

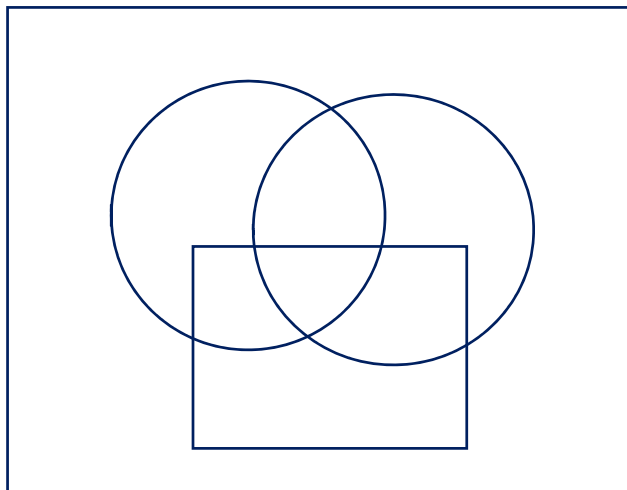
[12] 1. Evaluate each of the following. Show all work. Simplify your answers to a single integer.

a) $8! - 5!$

b) $C(8,4)$

c) $P(9,3)$

[12] 2. In a recent survey of 100 pet owners, it was found: 40 own a dog, 47 own a cat, and 36 own some type of reptile. It is also known that 16 own both cats and reptiles, 15 own cats and dogs, 13 own a dog and a reptile, while 7 own all three types of these pets.



a) How many pet owners own a cat and do not own a dog?

b) How many owners own only two of the three types of pets?

In problems 3, 4 and 5 you may leave your answers in terms of products, powers, permutations and/or combinations. You need not simplify your answers.

- [10] 3. A computer passcode is three digits followed by four letters from the alphabet. Letters may be or may not be capitalized. The computer is case sensitive and does not consider lower case and capital letters the same. How many passcodes are possible if
- a) repetition is not allowed?

 - b) digits cannot be repeated and letters may be repeated?
- [15] 4. An archer, when shooting an arrow at a given target hits the target 68% of the time. He enters a contest where he shoots 12 arrows at a given target.
- a) Find the probability he misses every shot.

 - b) Find the probability he hits exactly seven shots.

 - c) A prize of \$100 is given to the winner of the contest. If he is the last person to shoot and the highest score of any other archer is 10 hits out of twelve, what is the probability he wins the \$100 outright(i.e. no ties)?

[15] 5. A hand of 6 cards is dealt from a regular 52 card deck. Find the probability that

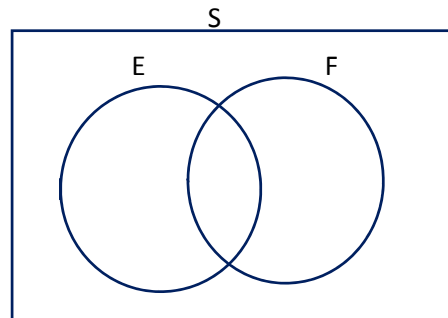
a) the hand contains no red cards.

b) the hand contains the queen of hearts and the jack of diamonds.

c) the hand contains all four jacks and at least 1 queen.

[15] 6. Let E and F be events of sample space S . Let $P(E) = 0.3$, $P(F) = 0.6$ and the $P(E \cup F) = 0.7$.

a) Fill in all probabilities in the Venn diagram shown.



b) Find $P(E \cap \bar{F})$.

c) Find $P(E \cap F)$.

d) Find the $P(E|F)$.

e) Are E and F independent events? Justify your answer.

- [12] 7. A manufacturer gets 70% of his supplies from supplier A and the remainder from supplier B. It is known that 8% of supplier A's supplies are defective and 6% of supplier B's are defective.
- Construct a tree diagram representing this data. Be sure to list all outcomes and their probabilities.
 - If a supply is found to be defective, find the probability it came from supplier B. Give your answer as an exact fraction which is reduced.
- [10] 8. a) Find the equation of the line through the points $(3,1)$ and $(-2,3)$. Give your answer in slope intercept form.
- Are the lines with equations $2x - 3y = -12$ and $y = \frac{2}{3}x + 4$ parallel, intersecting, or coincident? Justify your answer.

- [7] 9. How many different arrangements to all of the letters in REGRESS? Show all work and give a single integer as your answer.

[12] 10. Let $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 0 & -2 \\ 1 & 3 & -1 \end{bmatrix}$.

a) Find A^{-1} the inverse of matrix A .

b) Find AB if it is possible.

- [16] 11. Solve the following system of equations using reduced row-echelon form. Label all row operations and clearly state your final solution.

$$2x + 3y + 5z = 32$$

$$x + 4z = 28$$

$$3x + 2y + z = 14$$

- [12] 12. The following augmented matrices represent systems of equations in terms of x, y , and z . For each case state the general solution or that no solution exists.

a)
$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 1 & 1 & 1 & 12 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

b)
$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 8 \\ 0 & 1 & 0 & 6 \end{array} \right)$$

c)
$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \end{array} \right)$$

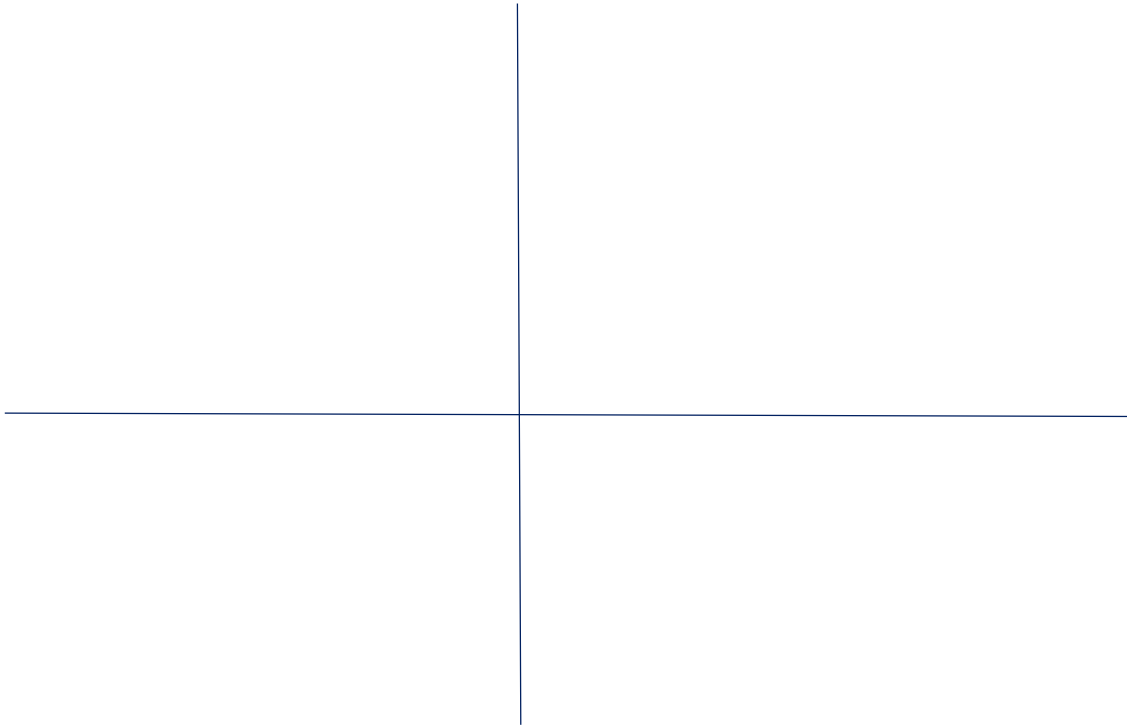
- [12] 13. Formulate an LP model for the following problem. (DO NOT ATTEMPT TO SOLVE IT.)

ACME manufacturing makes three types of road-runner traps. Blue traps require four springs, 12 units of metal and three hours of labor. Red traps use only 10 units of metal, two less springs than blue traps and also three hours of labor. Green traps require five springs, 13 units of metal, and one more hour of labor than a red trap. Each week ACME has 3300 units of metal available and 1850 springs. In addition ACME has a labor contract with Coyotes Local Union 101 requiring that they will provide at least 15 hours of labor for each of 12 employees. Due to costs ACME cannot provide more than 300 hours of total labor each week. If the profit on green traps is \$35, for blue traps \$17 and \$12 for red traps, how many of each trap should ACME manufacture to maximize their profit?

[12] 14. Consider the following linear programming problem.

$$\begin{aligned} &\text{Maximize } P = 4x + 2y \text{ subject to the constraints:} \\ &x \geq 0, \quad y \geq 0, \quad y \leq 6, \quad 3x - y \leq 30, \quad x + y \geq 8 \end{aligned}$$

a) Sketch the feasible region and label the corner points. Be sure to shade the feasible region.



b) Solve the problem geometrically and report the complete solution.

[12] 15. For each of the following tableaux, do one of the following.

- 1) If a pivot is required, write pivot needed, circle the pivot element BUT DO NOT PIVOT.
- 2) If there is no optimal solution say so and state why.
- 3) If the problem is finished report the complete solution.

a)

BV	P	x_1	x_2	s_1	s_2	RHS
x_2	0	1	4	1	0	11
s_2	0	0	2	-1	1	3
P	1	0	1	3	0	19

b)

BV	P	x_1	x_2	x_3	s_1	s_2	s_3	RHS
s_1	0	1	-2	1	1	0	0	12
s_2	0	2	-1	3	0	1	0	10
s_3	0	4	0	-2	0	0	1	6
P	1	2	-3	-2	0	0	0	0

c)

BV	P	x_1	x_2	x_3	s_1	s_2	s_3	RHS
x_1	0	1	4	2	1	0	0	11
s_2	0	0	3	5	0	2	1	13
s_3	0	0	2	1	0	1	1	7
P	1	0	-3	-1	1	2	0	12

- [16] 16. Use the simplex method to solve the following LP model. The MAXIMUM number of pivots you may need to make is 2.

Maximize $P = 3x_1 + 4x_2 + 2x_3$ subject to constraints

$$3x_1 + x_2 + 3x_3 \leq 6$$

$$x_1 - x_2 + 6x_3 \leq 12$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$