

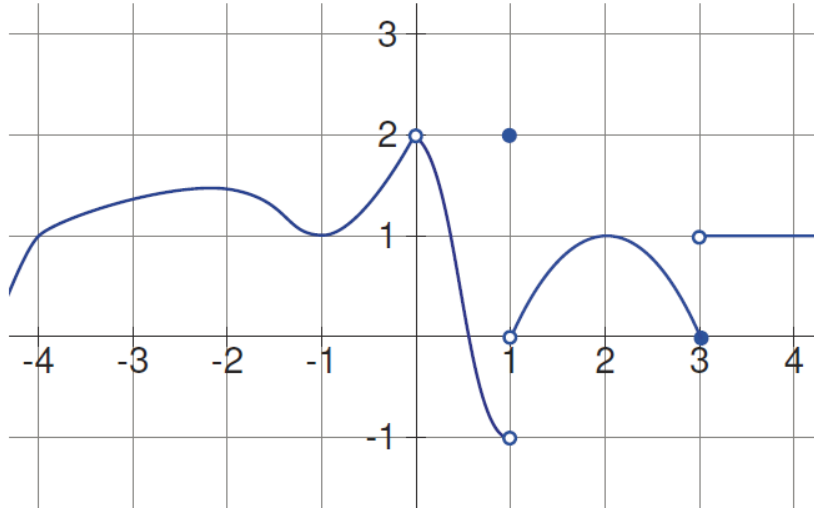
(15) 1. Calculate the following limits.

(a) 
$$\lim_{x \rightarrow -1} \frac{x}{\sqrt{x^2 - x}}$$

(b) 
$$\lim_{x \rightarrow \infty} \frac{-x^3 + x^2 - 10x + 11}{-x^3 - 7x^2 - 5}$$

(c) 
$$\lim_{x \rightarrow 4} \frac{x^2 + x - 20}{x^2 - 10x + 24}$$

(16) 2. Suppose that the graph of  $y = f(x)$  is as given below. Use the graph to find the following limits. If a limit does not exist, write “DNE”.



(a)  $\lim_{x \rightarrow 1^+} f(x) =$

(c)  $\lim_{x \rightarrow 2} f(x) =$

(b)  $\lim_{x \rightarrow 3} f(x) =$

(d)  $\lim_{x \rightarrow -1^-} f(x) =$

(28) 3. Compute the derivative of the following functions. **Do not** simplify.

(a)  $f(x) = \frac{4\sqrt{x} + 3x + 1}{x^2 - 3}$

(b)  $f(x) = (x^2 + 1)\ln(3x^2 + x)$

(c)  $f(x) = \frac{e^{-x^3}}{x^4}$

(d)  $f(x) = \sqrt[3]{x^3 + 12x - 1}$

(10) 4. Find the equation of the tangent line to the graph of the function

$$f(x) = \frac{2}{x+1} \text{ where } x=1. \text{ State your final answer in slope-intercept form.}$$

(12) 5. The revenue function  $R(x)$ , in thousands of dollars, from the sale of  $x$  items is given by

$$R(x) = 1200\sqrt{x^2 - 0.1x}.$$

Find the rate at which  $R(x)$  is changing when 30 units have been sold.

(16) 6. Given the two functions:

$$f(x) = -2x^2 + 3x \quad \text{and} \quad g(x) = x^3$$

(a) Find the ordered pairs where  $f$  and  $g$  intersect.

(b) Find the area bounded by the graphs of  $f$  and  $g$ . (*Hint: Draw a sketch first.*)

(14) 7. Suppose the *DERIVATIVE* of a certain function,  $f(x)$ , is given by

$$f'(x) = \frac{(x-1)^2}{x(x-3)}.$$

(a) Give the intervals over which  $f$  is increasing.

(b) Give the intervals over which  $f$  is decreasing.

(28) 8. Compute the following integrals.

(a)  $\int \left( 5x^{1/2} + 10 - \frac{4}{x^3} \right) dx$

(d)  $\int \frac{dx}{x \ln x}$

(c)  $\int_0^4 \frac{x}{\sqrt{x^2 + 9}} dx$

(d)  $\int x e^{-2x} dx$  (Use integration by parts.)

(10) 9. Find the producers' surplus at a price level of  $x=\$55$  for the Supply function

$$S(x) = 15 + 0.1x + 0.003x^2 .$$

(15)10. Let

$$f(x, y) = -6x^2y + 3x^5 + \frac{x^3}{y} + e^{y^2} . \text{ Find:}$$

(a)  $f_y$

(b)  $f_{yx}$

(c)  $f_{yx}(2, -1)$



(12)11. Find and identify the absolute minimum and maximum values of the function

$$f(x) = x^4 - 8x^2 - 4$$

on the interval  $[-1, 3]$ . Give both coordinates.

(12)12. Let

$$f(x, y) = 2y^3 - 6xy - x^2.$$

The critical points of  $f(x, y)$  are  $(0, 0)$ ,  $(9, -3)$ . Identify each critical point as a relative minimum, a relative maximum, or a saddle point.

- (12)13. A manufacturing firm has budgeted \$60,000 per month for labor and materials. If \$ $x$  thousand is spent on labor and \$ $y$  thousand is spent on materials, and if the monthly output (in units) is given by

$$N(x, y) = 4xy - 8x,$$

then how should the \$60,000 be allocated to labor and materials in order to maximize  $N$ ? Use the method of Lagrange multipliers. What is the maximum  $N$ ?