(28) 1. Compute the derivative of the following functions. Do not simplify.

(a) \( f(x) = -3x^4 + 6\sqrt{x} + 10 \)

(b) \( f(x) = (x^2 - 2x) \ln(x^2 + 1) \)

(c) \( f(x) = \frac{e^{x+1}}{e^{x-1}} \)

(d) \( f(x) = \sqrt{10 + x^3} \)
(10) 2. Calculate the following limits.

(a) \( \lim_{x \to -1} \frac{x^2 + 3x - 4}{x^2 - x} \)

(b) \( \lim_{x \to \infty} \frac{1 + x - x^3}{8x^4 - 5} \)

(16) 3. Suppose that the graph of \( y = f(x) \) is as given below. Use the graph to find the following limits. If a limit does not exist, write “DNE”.

(a) \( \lim_{x \to -3^+} f(x) = \)

(b) \( \lim_{x \to -3^-} f(x) = \)

(c) \( \lim_{x \to 2} f(x) = \)

(d) Is \( f \) continuous at \( x = -1 \)?

**There is a vertical asymptote at \( x = -1 \).**
(12) 4. Find the equation of the tangent line to the graph of the function
\[ f(x) = \frac{3}{x-1} \]
at the point (2,3). State your answer in slope-intercept form.

(12) 5. Ultra Mobile has costs that are given by
\[ C(x) = 2000 + 90x + 0.2x^2 \]
and price given by
\[ p(x) = 210 - 0.3x \]
where \( x \) is the number of SIM cards produced. What is the number of SIM cards that Ultra Mobile must produce and sell in order to maximize profit? (Recall that the revenue is \( R(x) = px \).) You must prove this using a derivative test.
(20) 6. Given the function \( f(x) = \frac{3x}{x+1} \)

(a) State the domain of \( f \).

(b) Find the vertical asymptote(s).

(c) Find the horizontal asymptote(s).

(d) Give the intervals over which \( f \) is increasing.

(e) Give the intervals over which \( f \) is concave up.

(f) Sketch on the above axes the graph of \( f(x) \). Draw the asymptotes as dashed lines. Mark the \( x \)- and \( y \)-intercepts. Plot additional points as needed.
(21) 7. Compute the following integrals.

(a) \[ \int \left( 4x^{1/2} + \frac{3}{x^2} - 12x \right) \, dx \]

(b) \[ \int xe^{-3x} \, dx \] (Use integration by parts.)

(c) \[ \int_{2}^{5} \frac{2x}{\sqrt{x^2+12}} \, dx \]
Given the two functions:
\[ f(x) = x^2 - 3x \quad \text{and} \quad g(x) = x \]

(a) Find the ordered pairs where \( f \) and \( g \) intersect.

(b) Find the area bounded by the graphs of \( f \) and \( g \). (Hint: Draw a sketch first.)
(12) 9. According to the U.S. Census Bureau, the population of the United States can be approximated by

\[ P(t) = 282.3e^{0.01t} \]

where \( P \) is in millions and \( t \) is the number of years since 2000.

Find the average value of the population from 2002 to 2006 (i.e. from \( t = 2 \) to \( t = 6 \)).

(15) 10. Let

\[ f(x, y) = -x^2y - 4x^4 + \frac{x}{y} \]

Find:

(a) \( f_y \)

(b) \( f_{yx} \)

(c) \( f_{yx}(1, 2) \)
(12) 11. Find and identify the absolute minimum and maximum values of the function
\[ f(x) = x^3 - 2x^2 - 4x + 4 \]
on the interval [0, 3]. Give both coordinates.

(12) 12. Let
\[ f(x, y) = x^3 - 3xy + y^3. \]
The critical points of \( f(x, y) \) are (0, 0), (1, 1). Identify each critical point as a relative minimum, a relative maximum, or a saddle point, showing work using the \( D \)-test.
(14) 13. Suppose $x$ TV’s are produced at one factory, and $y$ TV’s are produced at a second factory.

Use the method of Lagrange multipliers to find the minimum value of the company’s cost function

$$C(x, y) = 6x^2 + 12y^2$$

subject to the constraint that 90 TV’s are produced total, i.e. $x + y = 90$. 