Fall 2017–Final Exam
Math 150/151

[15] 1. Find the following limits. Show your work.

a) $\lim_{x\to 4} \dfrac{\sqrt{2x+1}-3}{x-4}$

b) $\lim_{\theta\to 0} \dfrac{\sin^2 5\theta}{\theta^2}$

c) $\lim_{x\to\infty} \dfrac{2-3x^5}{7x^5-6x^3}$

[10] 2. Use the definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{1}{x+1}$.

[40] 3. Find the derivative of each function. Do not simplify your answers.

a) $f(x) = xe^{-x^2}$

b) $p(x) = (1-3x)^5\sqrt{x^2+9}$

c) $F(x) = \displaystyle\int_1^{e^x} \sin(\ln t)\,dt$

d) $q(x) = \dfrac{2\sin x}{3+\cos x}$

e) $g(x) = x^{\sec x}$

[10] 4. Use implicit differentiation to find dy/dx if $\tan(x+y) = x^3y + 3y^2$.

[10] 5. Find an equation of the line tangent to the curve $y = 3\ln(x-1)+1$ at $x = 2$.

[10] 6. A boat is pulled into a dock by a rope attached to the bow of the boat and passing though a pulley on the dock that is 5 ft higher than the bow of the boat. If the rope is pulled in at a rate of 3 ft/s, how fast is the boat approaching the dock when it is 12 ft from the dock?

[20] 7. Let $f(x) = (x+2)^2 e^{-x}$.
Then $f'(x) = -(x^2+2x)e^{-x}$ and $f''(x) = (x^2-2)e^{-x}$.

a) Find the open intervals on which $f(x)$ is increasing and those on which $f(x)$ is decreasing.

b) Find all points on the graph of the function (x and y values) where $f(x)$ has local maxima and minima.

c) Find the open intervals on which the graph of $y = f(x)$ is concave up and those on which it is concave down.

d) Find all inflection points (x and y values) on the graph of $y = f(x)$.

[10] 8. Find the absolute maximum and absolute minimum values of the function $f(x) = x\sqrt{16-x^2}$ on the interval $[0, 3]$.

[10] 9. A cylindrical barrel is to be made with no top. The material for the side of the barrel costs \$1 per square foot. The material for the bottom costs \$3 per square foot. Find the dimensions of the barrel that minimize the cost of materials if the barrel must hold a volume of 6π cubic feet.

[27] 10. Evaluate the following indefinite integrals:

a) $\displaystyle\int \frac{2x^2 - 7x + 3}{\sqrt{x}}\,dx$

b) $\displaystyle\int \frac{3x}{x^2+4}\,dx$

c) $\displaystyle\int \left(2\sin 3x + \sec^2 x\right)\,dx$

[18] 11. Evaluate the following definite integrals:

a) $\displaystyle\int_{-3}^{3} \sqrt{9-x^2}\,dx$

b) $\displaystyle\int_{0}^{3} x^2 e^{x^3/6}\,dx$

[20] 12. Let R be the region bounded by $y = x$ and $y = 5x - x^2$.

a) Find the area of the region R.

b) Set up, but do not evaluate, an integral that represents the volume of the solid generated by revolving the region R about the x-axis.

c) Set up, but do not evaluate, an integral that represents the volume of the solid generated by revolving the region R about the line $x = 5$.