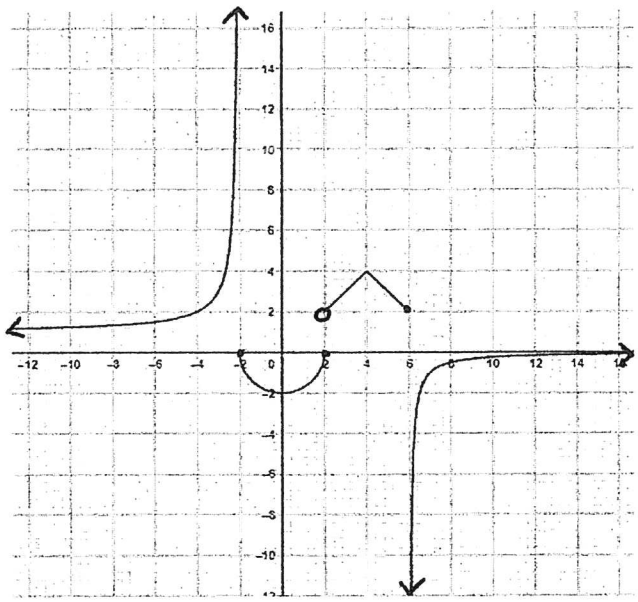
Finals Math150 Name:_____ Fall2021

1. (20 points) Consider the following graph of $f(x)$ to answer questions (a)-(j). If there is no limit then write DNE or $+\infty$ or $-\infty$ as appropriate.



(a) $\lim_{x\to -2^-} f(x) =$ _____

(b) $\lim_{x\to -2^+} f(x) =$ _____

(c) $\lim_{x\to -2} f(x) =$ _____

(d) $\lim_{x\to 2} f(x) =$ _____

(e) $\lim_{x\to -\infty} f(x) =$ _____

(f) $\lim_{x\to \infty} f(x) =$ _____

(g) $\lim_{x\to 4} f(x) =$ _____

(h) Find the x value(s) where the function is **continuous but not differentiable**. $x =$ _____

(i) Find $f(2) =$ _____ (j) Evaluate $\int_{-2}^{2} f(x)dx$ _____

2. (25 points) Compute the following limits algebraically. If the limits does not exist then write DNE or state clearly whether the limit is or $+\infty$ or $-\infty$.

a) $\lim_{x\to 7} \frac{3-\sqrt{x+2}}{7-x}$

b) $\lim_{x\to 3} \frac{9-x^2}{x^2-2x-3}$

c) $\lim_{x\to \infty} \frac{\sqrt{3x^2-2}}{2x}$

d) $\lim_{x\to \frac{\pi}{4}} (2\cos(x) + 5\sin(2x))$

(Give exact answer)

(e) $\lim_{x\to \infty} e^{-5t^2+1}$

3. (20points) Find the **DERIVATIVE** of the following functions: DO NOT need to simplify.

a) $f(x) = xe^{2x} + \ln(x^2) + \pi^2$

b) $f(x) = \sin(x^2) + \sin^2 x + \cos(3x)$

c) $F(x) = \int_7^{x^2} \sin(t^3)\ dt$

d) $f(x) = \frac{\sec x}{2+\cot x}$

e) $f(x) = (x^3 + 2\sqrt{x})(x^4 - 2)^{1/3}$

4. (6points) Use **definition** of the derivative (**any other method will receive zero points**) to find $f'(x)$ if

$f(x) = \frac{2}{x}$

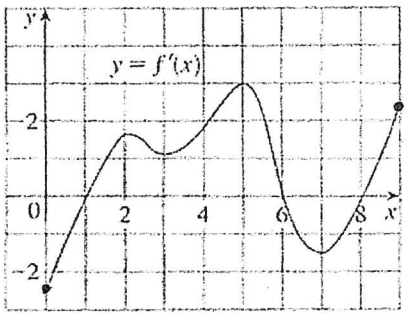
5. (7 points) Apply **logarithmic differentiation** to find the derivative $y = (\sin x)^{x^3}$

6. (7 points) Find the **equation of the tangent line** to the curve $y = \sin(\tan(2\,x))$ at $x = \pi$

7. (7 points) Suppose an inflating balloon is spherical in shape, and its radius is changing at the rate of 3 centimeters per second. At what rate is the volume changing when the diameter is 12 centimeters? Put units on answer. Do not use decimals. Exact answers only.

8. (6 points) Use **implicit differentiation** to find $\frac{dy}{dx}$ if $x^2y^3 + \sin x = 2x + 5y$

9. (12 points) Use the graph of the **DERIVATIVE, $f'(x)$,** to answer the following questions about $f(x)$. If there is more than one answer, then list all and separate them by comma, If the answer does not exists then enter DNE.



a) On what interval(s) is $f(x)$ **increasimg**?

b) At what value(s) of x does f have a local **minima**.
Min at x_____

c) On what interval(s) is $f(x)$**concave up**?

d) List the x-coordinates of any **inflection point(s)** of $f(x)$.

10. (8 points) Find the **absolute maximum and minimum** of $f(x) = e^{3x} - 4x$ on $[0,\ 2]$. Give exact answers.

11. (10 points) Consider $f(x) = x^3 - 3x^2 - 9x + 24$. Use calculus to determine the following:

a) Intervals where $f(x)$ is **increasing**:
b) Intervals where $f(x)$ is **decreasing**:

c) List the relative extrema in (x, y) form. Classify as max/min

12. (10 points) A rectangular area of 1600 ft² is to be fenced off. Two opposite sides will be fenced using fencing costing \$4 per linear foot and the other two opposing sides will be fenced using fencing costing \$2 per linear foot. Use calculus to find the dimensions of the rectangle of least cost.

13. (28 points) Evaluate the following indefinite integrals:

a) $\int \frac{2x^3+x^2+1}{\sqrt{x}}\,dx$

b) $\int (\sin(3x) + e^{3x})\,dx$

c) $\int x\sqrt{2x+1}\,dx$

d) $\int \frac{1+3x}{1+x^2}\,dx$

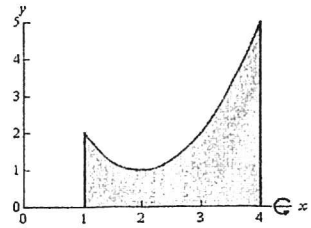
14. (14 points) Evaluate the following definite integrals:

a) $\int_e^{e^2} \frac{\ln x}{x}\,dx$

b) $\int_0^1 (1 + 2x^2)^2 dx$

15. (10 points) Find the **area** bounded by the two curves $f(x) = x^2 - 4$ and $g(x) = 2x - 1$. Sketch the region, shade the relevant area then find the area of this region.

16. (5 points) **SET UP, but do not evaluate an integral** that represents the volume of the solid generated by revolving the region bounded by $y = x^2 - 4x + 5$, $x = 1$, $x = 4$, and the x axis about the x axis.



17. (5 points) Let A be the region bounded by the curves $y = x$ and $y = x^2$ as shown by the picture.

SET UP, but do not evaluate an integral that represents the volume of the solid generated by revolving A about $y = -1$