1) Find the following limits. Show your work for credit. . If the limit does not exist, explain why. Do not use L'Hopital's rule. <5 each>

a) $\lim_{x\to3}\frac{\sqrt{x+1}-2}{x-3}$ $\frac{0}{0}$

$\lim_{x\to3}\frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}{(x-3)(\sqrt{x+1}+2)}$

$=\lim_{x\to3}\frac{x+1-4}{(x-3)(\sqrt{x+1}+2)} = \lim_{x\to3}\frac{\cancel{x-3}^{\,1}}{(\cancel{x-3})(\sqrt{x+1}+2)}$

$=\frac{1}{\sqrt{4}+2}=\frac{1}{2+2}=\boxed{\frac{1}{4}}$

b) $\lim_{x\to0}\frac{\sin(7x)}{\sin(3x)}$

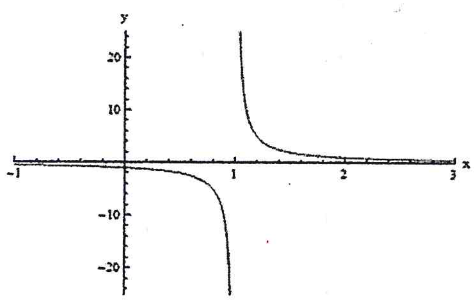
$\lim_{x\to0}\frac{\frac{\sin(7x)}{7x}\cdot 7x}{\frac{\sin(3x)}{3x}\cdot 3x}$

$=\frac{\lim_{x\to0}\frac{\sin(7x)}{7x}}{\lim_{x\to0}\frac{\sin(3x)}{3x}}\cdot\lim_{x\to0}\frac{7\cancel{x}}{3\cancel{x}}$

$=\frac{1}{1}\cdot\frac{7}{3}$

$=\boxed{7/3}$

c) Use the graph of y = f(x) to find the limits:



- $\lim_{x\to1^+} f(x) =$ ∞
- $\lim_{x\to1^-} f(x) =$ $-\infty$
- $\lim_{x\to1} f(x) =$ DNE

d) $\lim_{x\to-\infty}\frac{x^3-2x+1}{2-5x^3}$

$=\lim_{x\to-\infty}\frac{\frac{x^3}{x^3}-\frac{2x}{x^3}+\frac{1}{x^3}}{\frac{2}{x^3}-\frac{5x^3}{x^3}}$

$=\lim_{x\to-\infty}\frac{1-\cancel{\frac{2}{x^2}}^{\,0}+\cancel{\frac{1}{x^3}}^{\,0}}{\cancel{\frac{2}{x^3}}^{\,0}-5} = \boxed{\frac{1}{-5}}$

2) Find the equation of the tangent line to $f(x) = 2xe^x + 3$ when x=0

<10 pts>

$f' = 2e^x + 2xe^x$

$f'(0) = 2e^0 + 2(0)e^0$

$=2(1)+0=2$

$x=0,\ y=2(0)e^0+3=0+3=3$

$y-3=2(x-0)$

$\boxed{y=2x+3}$

3) The absolute extrema of f(x) = $\frac{\ln x}{x}$ on the interval $[1, e^2]$.

<10 pts>

$\boxed{\begin{array}{l}\text{abs max } (e, \frac{1}{e}) \\ \text{abs min } (1, 0)\end{array}}$

$f' = \frac{\frac{1}{x} \cdot x - 1 \ln x}{x^2}$

$= \frac{1 - \ln x}{x^2} = 0$

$\frac{1-\ln x}{x^2} = 0$

$1 - \ln x = 0$

$\ln x = 1$

$x = e$

$f(1) = \frac{\ln 1}{1} = 0$

$f(e) = \frac{\ln e}{e} = \frac{1}{e} \approx .37$

$f(e^2) = \frac{\ln(e^2)}{e^2} = \frac{2}{e^2} \approx .27$

f' is undef. at x = 0, But not in interval.

4) Use the definition to find the derivative of $f(x) = \frac{3}{x-2}$. Any other method will be worth very little. Do not skip any steps.

<10 pts>

$\lim_{h \to 0} \frac{\frac{3}{x+h-2} - \frac{3}{x-2}}{h}$

$= \lim_{h \to 0} \frac{3(x-2) - 3(x+h-2)}{(x+h-2)(x-2)} \cdot \frac{1}{h}$

$= \lim_{h \to 0} \frac{\cancel{3x} - \cancel{6} - \cancel{3x} - 3h + \cancel{6}}{(x+h-2)(x-2)} \cdot \frac{1}{h}$

$= \lim_{h \to 0} \frac{-3\cancel{h}}{\cancel{h}(x+h-2)(x-2)}$

$= \frac{-3}{(x+0-2)(x-2)} = \boxed{\frac{-3}{(x-2)^2}}$

5) Find dy/dx by implicit differentiation: $5x^2 - xy - 3y^4 = x^3$

<8 pts>

$10x - y - x\frac{dy}{dx} - 12y^3\frac{dy}{dx} = 3x^2$

$-x\frac{dy}{dx} - 12y^3\frac{dy}{dx} = 3x^2 - 10x + y +$

$\frac{dy}{dx}(-x - 12y^3) = 3x^2 - 10x + y$

$\boxed{\frac{dy}{dx} = \frac{3x^2 - 10x + y}{-x - 12y^3}}$

6) Differentiate the following. You don't need to simplify your answers. <7 each>

a) $f(t) = 3\tan(t^2) + 3t\ln(t)$

$$f'(t) = 3\sec^2(t^2)\cdot 2t + 3\ln(t) + \frac{1}{t}\cdot 3t$$

$5x^{1/2}$

b) b) $g(x) = (\sec(x) - 5\sqrt{x} + \pi)^{1/3}$

$$g' = \frac{1}{3}(\sec(x) - 5\sqrt{x} + \pi)^{-2/3}\left(\sec x\tan x - \frac{5}{2}x^{-1/2}\right)$$

c) $f(x) = (cosx)^{2x}$

$$y = (\cos x)^{2x}$$

$$\ln y = \ln(\cos x)^{2x}$$

$$\ln y = 2x\ln(\cos x)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = 2\ln(\cos x) + \frac{1}{\cos x}(-\sin x)(2)$$

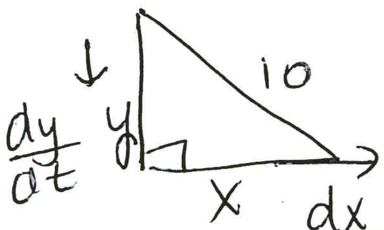
$$\frac{dy}{dx} = [2\ln(\cos x) - 2x\tan x)]\cdot y$$

$$= [2\ln(\cos x) - 2x\tan(x)]\,(\cos x)$$

d) $g(\theta) = \frac{sin^2\theta}{5\theta}$ $= \frac{(\sin\theta)^2}{5\theta}$

$$g' = \frac{2\sin\theta\cos\theta\cdot 5\theta - 5\sin^2\theta}{(5\theta)^2} = \frac{10\theta\sin\theta\cos\theta - 5\sin^2\theta}{25\theta^2}$$

7) A ladder 10 feet long is resting against a wall. If the bottom of the ladder is sliding away from the wall at a rate of 1 foot per second, how fast is the top of the ladder moving down when the bottom of the ladder is 8 feet from the wall? You must show all work clearly for credit. Put units on answer. <8 pts>

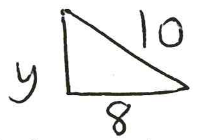


$\frac{dx}{dt} = 1 \frac{ft}{sec}$

$x^2 + y^2 = 10^2$

$2x\frac{dx}{dx} + 2y\frac{dy}{dt} = 0$

$2y\frac{dy}{dt} = -2x\frac{dx}{dt}$

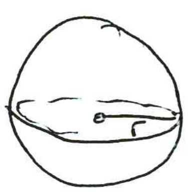


$y^2 + 8^2 = 10^2$

$y^2 = 36 \quad y = 6$

$\frac{dy}{dx} = -\frac{x}{y}\frac{dx}{dt} = -\frac{8}{6}(1) = \boxed{-\frac{4}{3}\frac{ft}{sec}}$

8) Suppose that an inflating balloon is spherical in shape, and its radius is changing at the rate of 3 centimeters per second. At what rate is the volume changing when the diameter is 20 centimeters? Put units on answer. Don't use decimals. <7 pts>



$d = 20$

$r = 10$

$V = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$= 4\pi(10)^2(3) = \boxed{1200\pi \text{ cm}^3/\text{sec}}$

9) Given $f(x) = 3x^5 - 5x^3 + 3$, use calculus to find the following. You must show all work. <9 pts>

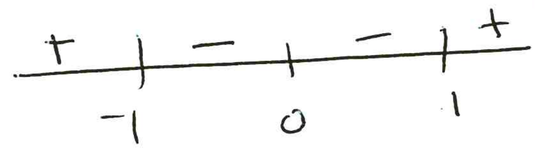
a) Find where f(x) is increasing and decreasing. Use interval notation.

$f' = 15x^4 - 15x^2$

$15x^4 - 15x^2 = 0$

$15x^2(x^2 - 1) = 0$

$x = 0, 1, -1$



$\boxed{\begin{array}{l} \text{incr } (-\infty, -1) \cup (1, \infty) \\ \text{decr } (-1, 0) \cup (0, 1) \end{array}}$

$f'(-2)$: $15(-2)^4 - 15(-2)^2$ $+$

$f'(-\frac{1}{2})$: $= 15(-\frac{1}{2})^4 - 15(-\frac{1}{2})^2$ $-$

$f'(\frac{1}{2})$: $15(\frac{1}{2})^4 - 15(\frac{1}{2})^2$ $-$

$f'(2)$: $15(2)^4 - 15(2)^2$ $+$

b) Find the relative extrema in (x,y) form. Classify as max or min.

$f(-1) = 3(-1)^5 - 5(-1)^3 + 3 = 5$ rel. max

$f(1) = 3(1)^5 - 5(1)^3 + 3 = 1$ rel. min

10) Find the inflection point(s) of $f(x) = xe^x$. Use exact values. <6 pts>

$f' = 1e^x + xe^x$

$f'' = e^x + 1e^x + xe^x$

$= 2e^x + xe^x$

$\underbrace{e^x}_{\text{never}=0}(2+x) = 0 \Rightarrow x = -2$

$-$ | $+$

$f''(-3)$ -2 $f''(3)$

$2e^{-3} - 3e^{-3}$ $\quad$ $2e^3 + 3e^3$

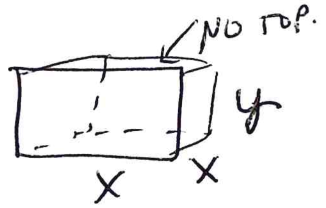
$-1e^{-3}$ $\quad$ $+$

$-$

$f(-2) = -2e^{-2}$

$\boxed{(-2, -2e^{-2})}$

11) An open (no lid) rectangular box with square base is to be made from 48 ft.^{2} of material. What dimensions will result in a box with the largest possible volume ? Show all work clearly using calculus for credit. Put units on your answers. <10 pts>



$\max V = x^2 y$

$48 = x^2 + 4xy$

$\frac{48 - x^2}{4x} = y$

$\max V = x^2\left(\frac{48 - x^2}{4x}\right)$

$V = 12x - \frac{x^3}{4}$

$V' = 12 - \frac{3}{4}x^2 = 0$

$12 = \frac{3}{4}x^2$

$16 = x^2 \Rightarrow x = \pm 4 \Rightarrow x = 4$

$+$ | $-$

$v'(3)$ 4 $v'(5)$

so max.

$x = 4$

$\Rightarrow y = \frac{48 - 4^2}{4(4)}$

$= 2$

$\boxed{4 \times 4 \times 2 \text{ ft ft ft}}$

12) Evaluate the integrals. <7 each>

a) $\int \frac{x^4 - 3x^2 + x}{x^2}\,dx$

$= \int (x^2 - 3 + x^{-1})\,dx$

$= \frac{x^3}{3} - 3x + \ln|x| + C$

b) $\int \; t\sqrt{5t+3}\,dt$

$u = 5t+3 \Rightarrow \frac{u-3}{5} = t$

$du = 5\,dt$

$\frac{du}{5} = dt$

$\int \frac{u-3}{5} \cdot u^{1/2} \frac{du}{5}$

$= \frac{1}{25}\left[u^{3/2} - 3u^{1/2}\right] du$

$= \frac{1}{25}\left[\frac{2u^{5/2}}{5} - \frac{2\cancel{3}u^{3/2}}{\cancel{3}}\right] + C$

$\boxed{= \frac{1}{25}\left[\frac{2}{5}(5t+3)^{5/2} - 2(5t+3)^{3/2}\right] + C}$

c) $\int (3sec^2 x - \pi + \frac{3}{1+x^2})dx$

$3\tan x - \pi x + 3\tan^{-1} x + C$

13) Evaluate. Show all work. **Leave in exact form (no decimals) <8 each>**

a) $\int_0^2 4xe^{x^2}\,dx$

$4\int_0^2 x e^{x^2} dx$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

Bounds:

$x = 0 \Rightarrow u = 0^2 = 0$

$x = 2 \Rightarrow u = 2^2 = 4$

$4\int_0^4 e^u \frac{du}{2} = 2\int_0^4 e^u du = 2e^u \Big|_0^4 = 2e^4 - 2e^0$

$\boxed{= 2e^4 - 2}$

b) $\int_0^{\frac{\pi}{3}}(4\sin x + 3cosx)\,dx$. Leave answer in exact form (no decimals)

$$= -4\cos x + 3\sin x \Big|_0^{\pi/3}$$

$$= \left[-4\cos\frac{\pi}{3} + 3\sin\frac{\pi}{3}\right] - \left[-4\cos 0 + 3\sin 0\right]$$

$$= -4\left(\frac{1}{2}\right) + 3\left(\frac{\sqrt{3}}{2}\right) - \left[-4(1) + 0\right] = -2 + \frac{3\sqrt{3}}{2} + 4$$

$$= 2 + \frac{3\sqrt{3}}{2}$$

14) Sketch the region bounded by $y = 2x^2 + 10$ and $y = 4x + 16$. Shade the region. Then, find the area of this region. <10 pts>

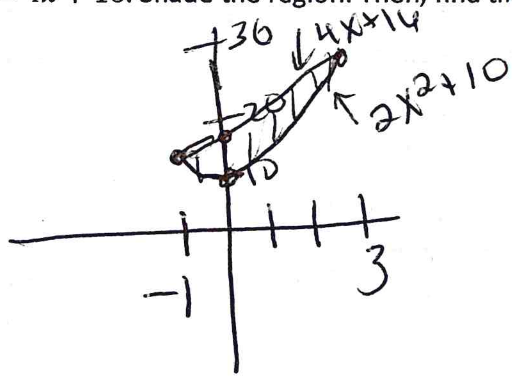
$2x^2 + 10 = 4x + 16$

$2x^2 - 4x - 6 = 0$

$2(x^2 - 2x - 3) = 0$

$2(x-3)(x+1) = 0$

$x = 3, -1.$



x	$y = 2x^2+10$
-1	12
0	10
3	28

x	$y = 4x+16$
-1	12
0	16
3	28

$$\int_{-1}^{3} (4x+16) - (2x^2+10)\,dx$$

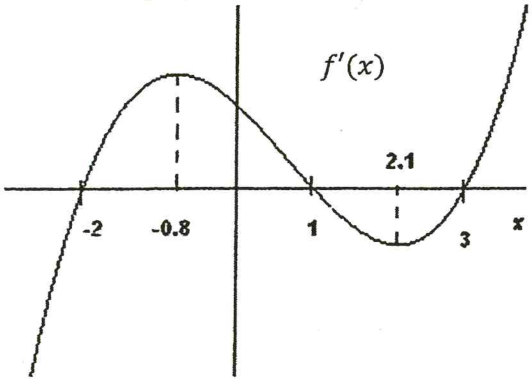
$$= \int_{-1}^{3} -2x^2 + 4x + 6\,dx$$

$$= -\frac{2x^3}{3} + 2x^2 + 6x \Big|_{-1}^{3}$$

$$= \left(\frac{-2(3)^3}{3} + 2(3)^2 + 6(3)\right) - \left(\frac{-2}{3}(-1)^3 + 2(-1)^2 + 6(-1)\right)$$

$$= \frac{64}{3}$$

15) Given the graph of the **DERIVATIVE**, f', answer the following: <12 pts>



a) On which interval(s) is f(x) increasing?

$(-2,1)\cup(3,\infty)$

b) On which interval(s) is f(x) concave up?

$(-\infty,-0.8)\cup(2.1,\infty)$

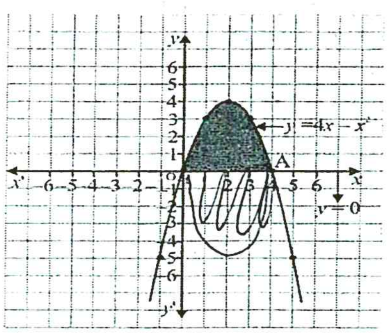
c) List the x-coordinates of any relative minima of f(x)

$x=-2,\ x=3$

d) List the x-coordinates of any inflection points of f(x

$x=-.8,\ x=2.1$

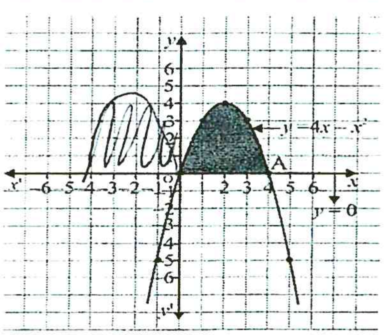
16) **<u>SET UP, but no do not evaluate an integral</u>** that represents the volume of the solid generated by revolving the region A below about the: <15 pts>



a) x –axis (again, set up integral only)

disks

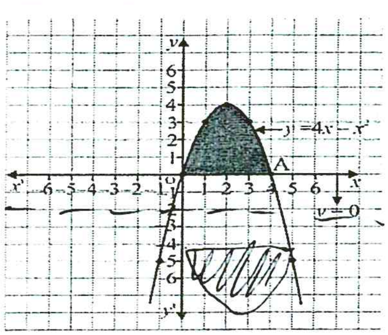
$$\int_0^4 \pi(4x-x^2)^2\,dx$$



b) y-axis (again, set up integral only)

Shells

$$\int_0^4 2\pi x(4x-x^2)\,dx$$



c) the line y= -2 (again, set up integral only)

washers

$$\int_0^4 \left(\pi(4x-x^2+2)^2-\pi(2)^2\right)dx$$