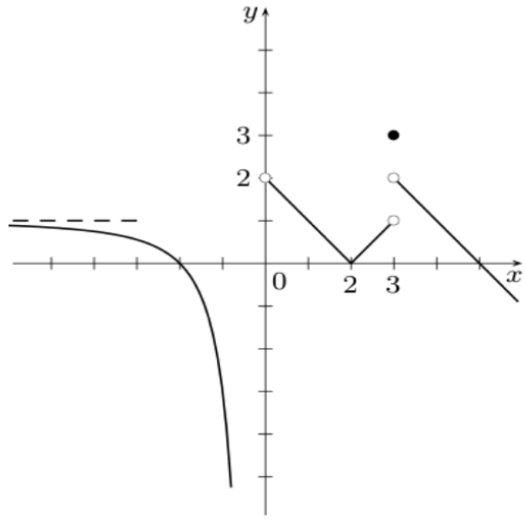
Finals Math150 Name:_____ Spring2022

1. (10 points) Consider the following graph of $f(x)$ to answer questions (a)-(i). If there is no limit then write DNE or $+\infty$ or $-\infty$ as appropriate.




(a) $f(0) =$ _____

(b) $f(2) =$ _____

(c) $f(3) =$ _____

(d) $\lim_{x\to 0^-} f(x) =$ _____

(e) $\lim_{x\to 0} f(x) =$ _____

(f) $\lim_{x\to 3^-} f(x) =$ _____

(g) $\lim_{x\to 3^+} f(x) =$ _____

(h) $\lim_{x\to -\infty} f(x) =$ _____

(i) List the $x$ values where $f$ is **<u>continuous but not differentiable</u>**: $x =$ _____

2. (15 points) Compute the following limits algebraically. If the limit does not exist then write DNE or state clearly whether the limit is or $+\infty$ or $-\infty$.

a) $\lim_{x\to 0} \frac{\sqrt{16+x}-4}{x}$

b) $\lim_{x\to \infty} \frac{\sqrt{81x^2+4x+9}}{3x}$

c) $\lim_{x\to -2} \frac{4-x^2}{x+2}$

3. (8 points) Find the derivative using the **<u>limit definition of derivative</u>**. $f(x) = -2x^2 + x$

4. (25points) Find the derivative of the following functions: **<u>DO NOT need to simplify</u>**.

a) $f(x) = x^2 e^{-x} + \ln(x^2) + e^2$

b) $f(x) = \tan^{-1}(3x) + \cos^2 x$

c) $f(x) = \frac{2x^2+1}{3x+\sec x}$

(d) $f(x) = \left(\sqrt{x} + \frac{1}{x}\right)(x^3 + 5)^{2/3}$

(e) $F(x) = \int_1^{e^{x^2}} \sin(\ln t)\, dt$

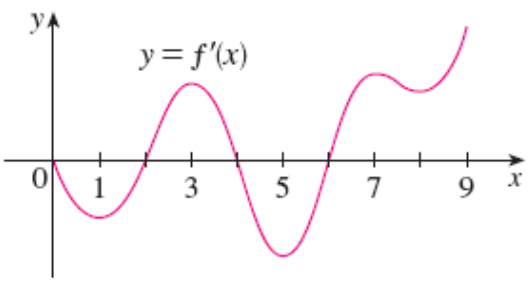
5. (8 points) Apply **<u>logarithmic differentiation</u>** to find the derivative $y = (\tan x)^{\cos x}$

6. (6 points) Find the **<u>equation of the tangent line</u>** to the curve $y = x\cos x + \sin x$ at $x = \pi$

7. (8 points) Use **<u>implicit differentiation</u>** to find $\frac{dy}{dx}$ if $y\cos x = 3x^2 + 5y^2$

8. (8 points) The radius of a circular oilslick expands at a rate of 2 m/min. How fast is the area of the oil slick increasing when the radius is 5 m? Put units on your answer.

9. (12 points) Use the graph of the **DERIVATIVE, $f'(x)$,** to answer the following questions about $f(x)$. (Assume the function is defined for $0 \leq x \leq 9$) If there is more than one answer list the, all and separate them as coma, If the answer does not exists then enter DNE.




a) On what interval(s) is $f(x)$ increasing?

b) At what value(s) of $x$ does $f$ have a local minima.

c) On what interval(s) is $f(x)$concave up?

d) List the $x$-coordinates of any inflection point(s) of $f(x)$.

10. (12 points) Consider $f(x) = -x^3 + 9x^2 - 15x - 12$ Use calculus to determine the following:

a) Intervals where $f(x)$ is **<u>concave up</u>**: Give exact answers. _____

b) Intervals where $f(x)$ is **<u>concave down</u>**: Give exact answers. _____

c) List the **<u>inflection point(s)</u>** as an ordered pair $(x, y)$_____

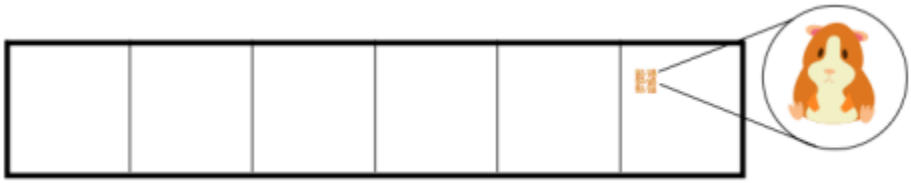
11. (12 points) Consider $f(x) = x^5 \ln x$ Use calculus to determine the following:

a) Intervals where $f(x)$ is **increasing**: Give exact answers. _____

b) Intervals where $f(x)$ is **decreasing**: Give exact answers. _____

c) List the relative extrema in $(x, y)$ form. Classify as max/min _____

12. (10 points) Jimmy is constructing a hamster pen to breed his furry friends on the farm. The construction (see picture below) will involve dividing the total area into pens of equal sizes as shown below. The total area is 60 square feet. The cost of outside fencing is $10 per foot. The interior fencing is $5 per foot. Find the dimensions to minimize the cost of fencing. You must verify your answer by using method of calculus. Put units in your final answer.



13. (28 points) Evaluate the following indefinite integrals:

a) $\int \left(5x^2 + \pi + \frac{1}{x}\right) dx$

b) $\int (sec^2(2x) + e^{2x})\, dx$

c) $\int x\sqrt{2x+3}\, dx$

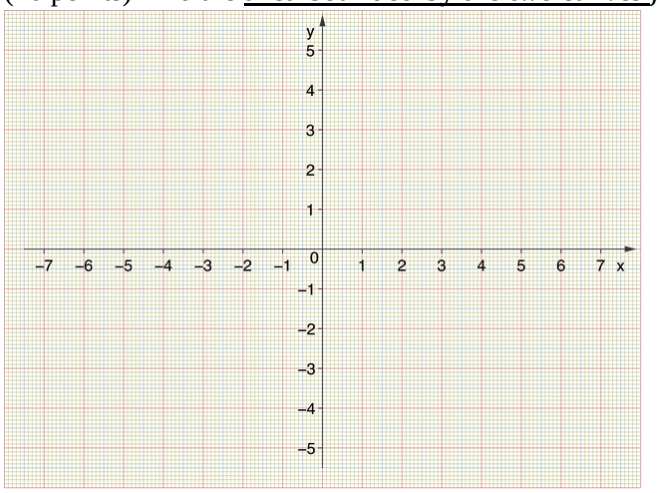
d) $\int 2xe^{3x^2}\, dx$

14. (16 points) Evaluate the following definite integrals:

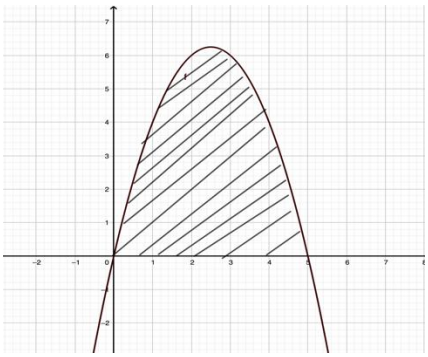
a) $\int_0^1 \frac{1+2x}{1+x^2}\, dx$

b) $\int_1^4 \frac{\sqrt{x}-x}{x^2}\, dx$

15. (10 points) Find the **<u>area bounded by the two curves</u>** $f(x) = x^2 - 2x + 3$ and $g(x) = 2x$. Sketch the region, shade the relevant area then find the area of this region.

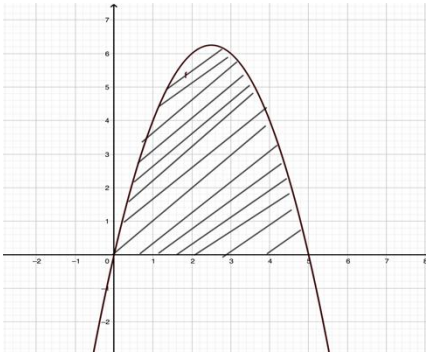


16. ( 4points each) **<u>SET UP, but do not evaluate an integral</u>** that represents the volume of the solid generated by revolving the region as shown in the picture bounded by the curve $y = 5x - x^2$ and the $x$-axis about



a) $x$ – axis.

b) $y$- axis



c) $y = -2$