1. Calculate the following integrals.

[8] a) \( \int \frac{\ln x}{\sqrt{x}} \, dx = \)

[8] b) \( \int \frac{2x + 3}{x^2 + 2x + 1} \, dx \)

[8] c) \( \int \tan^3 x \sec^4 x \, dx \)
d) \[ \int \frac{\sqrt{x^2 - 36}}{x} \, dx \]

e) Use the trapezoidal rule to express the integral below as a numerical sum. Use step \( \Delta x = 0.5 \)

(No need to evaluate the sum.) \[ \int_{0}^{2} \frac{x}{9x^3 + 1} \, dx \]

2. Calculate the following definite integrals.

a) \[ \int_{0}^{\infty} x^2 e^{-x^3} \, dx \]

b) \[ \int_{1}^{2} \frac{dx}{\sqrt{x - 1}} \]
3. Find the following limits.

[10] a) \( \lim_{x \to 0^+} \left( e^{3x} - e^x \right) \cot x \)

[10] b) \( \lim_{x \to 0^+} (2x + 1)^{1/x} \)

[8] 4. Evaluate the series or, if it is divergent, explain why.

\[
\sum_{n=2}^{\infty} \left( \frac{e}{\pi} \right)^{n-1}
\]
5. Determine if the series is absolutely convergent, conditionally convergent or divergent. Include your arguments and reasoning to receive credit for your answer.

[a] \[ \sum_{n=0}^{\infty} \left( \frac{3n + 2}{5n + 3} \right)^n \]

[b] \[ \sum_{n=0}^{\infty} \frac{(-7)^n}{3n - 1} \]

[c] \[ \sum_{n=0}^{\infty} \frac{(-2)^n n^3}{n!} \]
6. Find the interval of convergence of the series. Include the analysis of the endpoints.

\[
\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{3^n (n + 1)}
\]

[8]  b) If you use the first 4 terms to approximate the value of this series for \( x = \frac{1}{2} \), estimate how large the error could be. Explain your answer.

[8]  7. Find MacLaurin expansion of the function

\[
f(x) = \frac{x^2}{1 + \pi x}
\]
8. [8] a) Represent the function $f(x) = \arctan(x)$ (the same as $f(x) = \tan^{-1}(x)$) as a power series.

[8] b) Use your result to find the exact value of

$$\sum_{n=0}^{\infty} (-1)^n \frac{3^{(2n+1)/2}}{2n + 1}$$
9. Write the first 4 non-zero terms of the Taylor expansion of the function $f(x) = \cos(x)$ around point $a = \pi/4$.

10. Find the area enclosed by the smaller loop made by the polar curve $r = 1 + 2\cos \theta$. 

![Graph of the polar curve $r = 1 + 2\cos \theta$.]
11.  [8]  a) Find the arc length of the part of the curve given below for $0 \leq t \leq \ln 2$.

\[
\begin{align*}
\begin{aligned}
 x &= \frac{2}{3} e^{3t/2} \\
y &= e^t
\end{aligned}
\end{align*}
\]

[8]  b) Find the equation of the line tangent to the above curve at $t = 0$. 


12. [8] a) Determine the eccentricity and identify the conic as one of four types: circle, parabola, hyperbola, or ellipse.

(i) \( r = \frac{5}{6 - 3\sin \theta} \)

(ii) \( r = \frac{6}{1 + 2\cos \theta} \)

(eccentricity) \( e = \)

name: ___________

[8] b) Sketch the second curve (ii) to all that apply, label by coordinates \textbf{intercepts} (intersections with \( x \) and \( y \) axes), the \textbf{center} and the \textbf{foci}. Draw one directrix. Then fill in the blanks at the bottom of the page:

Work:

<table>
<thead>
<tr>
<th>Answers to all that apply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertices:</td>
</tr>
<tr>
<td>Center:</td>
</tr>
<tr>
<td>Foci:</td>
</tr>
<tr>
<td>Equation of directrix:</td>
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<tr>
<td>The main axes: ( a = ) ( b = )</td>
</tr>
</tbody>
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