1. (18) Find each of the following limits. Give clear reasons for your answers.

(a) $\displaystyle\lim_{x\to 0} \frac{x - \sin x}{x^3}$

(b) $\displaystyle\lim_{x\to\infty} \sec\left(\frac{x^2+1}{x^3}\right)$

(c) $\displaystyle\lim_{x\to 0^+} (\cos x)^{1/x^2}$

2. (30) Evaluate each of the following indefinite integrals.

(a) $\int x^2 \ln x \, dx$

(b) $\int \sqrt{25 - x^2} \, dx$

(c) $\int \frac{dx}{x(x^2 - 16)}$

3. (30) Evaluate the following definite integrals. Otherwise, show that it diverges. Note: Some of these integrals may be improper.

(a) $\displaystyle\int_0^{\pi/4} \cos^3 x\,dx$

(b) $\displaystyle\int_0^1 \frac{dx}{(x-1)^2}$

(c) $\displaystyle\int_0^\infty \frac{dx}{1+x^2}$

4. (32) Determine whether the following series converge or diverge. Give a clear reason for each answer. Name any tests you are using.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{6n^2-6}{n^2+n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^n n!}$

(d) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{-3}{2}\right)^n$

5. (12) Determine whether the following series is absolutely convergent, conditionally convergent, or divergent. Give reasons for your answer.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

6. (12) Find the interval of convergence for the power series. Be sure to check the end points.

$$\sum_{n=1}^{\infty} \frac{(-1)^n (2x-1)^n}{n^3}$$

7. (8) Find the Taylor polynomial of degree 3 for

$$f(x) = \cos x \quad \text{about} \quad a = \frac{\pi}{4}.$$

8. (10) Determine the Maclaurin series for each of the following. You may use any technique you like to find the series, which includes quoting the Maclaurin series for well-known functions. Show all work. (Recall that the Maclaurin series for a function is the Taylor series at $a = 0$.)

   (a) $f(x) = x^2 \sin x$.

   (b) $f(x) = \dfrac{x^4}{2 - x}$.

9. (16) A curve has the parametric equations

$$x = \cos t, \qquad y = 1 + \sin t, \qquad 0 \le t \le 2\pi.$$

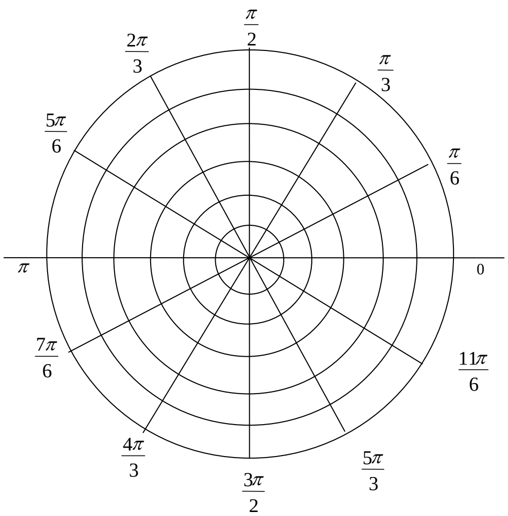
(a) Find $\dfrac{dy}{dx}$ when $t = \dfrac{\pi}{4}$.

(b) Find the equation of the line tangent to the curve at $t = \dfrac{\pi}{4}$. Write it in $y = mx + b$ form.

(c) Eliminate the parameter to find a Cartesian (x, y) equation of the curve.

(d) Using (c), or otherwise, identify the curve.

10. (12) Use integration, in polar coordinates, to compute the area of the region that is inside **one leaf** of the curve $r = \sin(3\theta)$. You may find the following grid useful.



11. (10) Find the length of the curve

$$x = e^t(\sin t - \cos t), \qquad y = e^t(\sin t + \cos t), \qquad 0 \le t \le 1.$$

12. (10) Estimate the value of the following integral to within 0.001. You may leave your answer as a sum and difference of fractions.

$$\int_0^1 e^{-x^2}\,dx$$