Math 125 Final Exam, Spring 2017

- The following exam has 4 parts, 21 problems, and 8 pages. Please stop and make sure that your exam has all its pages.
- Please raise your hand if you have any questions or need a restroom break.
- When you have completed your exam, raise your hand and the instructor will collect your exam. DO NOT begin packing up until you have turned your exam in.
- ANY use of cell phones or electronics other than an appropriate calculator will result in you receiving a zero on your final exam.
- ANY cheating (cheat sheets, communicating with classmates, etc.) will result in you receiving a zero on your final exam.

Section 1: Quick problems. Show work to receive partial credit. Make sure you simplify fully and round appropriately.

1. [5] Complete the indicated operation and give your answer in scientific notation. Round appropriately as your final step:
   
   a. \( \frac{1 \times 10^4 + 6.2 \times 10^6}{1000 + 6,200,000} = \frac{6,201,000}{6.2 \times 10^6} \)
   
   \[ \text{Rounding} \rightarrow 6.2 \times 10^6 \]

2. [5] Simplify. Express results with positive exponents only:
   
   a. \( \left( \frac{-2x^3}{y^{-7}} \right)^4 = \frac{(-2)^4 (x^3)^4}{(y^{-7})^4} = \frac{16x^{12}}{y^{-28}} = 16x^{12}y^{28} \)

3. [7] Perform the indicated operations and simplify. Be sure to write your answer in the correct form:
   
   a. \( (12x - 9x^2 + 4) \div (3x - 1) \)

   \[
   \begin{align*}
   &\frac{-3x + 3}{3x - 1} - \frac{-9x^2 + 12x + 9}{-9x^2 + 3x} \\
   &- \left( \frac{-9x^2 + 3x}{9x + 4} \right) \\
   &- \left( \frac{9x - 3}{7} \right) \\
   &= -3x + 3 + \frac{7}{3x - 1}
   \end{align*}
   \]
4. [15] Factor each polynomial completely:
   a. $7x^2 - 23x + 6$
      
      $\boxed{(7x - 2)(x - 3)}$
      
      check $7x^2 - 2x - 21x + 6$
      $= 7x^2 - 23x + 6$
      
   b. $x^4 - 81$
      
      $\frac{(x^2 - 9)(x^2 + 9)}{(x - 3)(x + 3)(x^2 + 9)}$
      
   c. $27x^3 + 1$
      
      $\boxed{(3x + 1)(9x^2 - 3x + 1)}$
      
      check $27x^3 - 9x^2 + 3x + 9x^2 - 3x + 1$
      $= 27x^3 + 1$

5. [5] Perform the indicated operation and simplify:
   a. $(1 - 5i) - (3i - 4)$
      
      $1 - 5i - 3i + 4 = \boxed{5 - 8i}$

6. [5] Simplify and solve for x. Give an exact answer:
   a. $x = \log_2 \frac{1}{4}$
      
      $2^x = \frac{1}{4}$
      
      $2^{-2} = \frac{1}{4}$
      
      $x = -2$
Section 2: Graphing and calculations. Show all work to receive credit. Be sure to simplify.

7. [9] Considering the following function: \( f(x) = \sqrt{3 - 2x} \)
   a. Find the domain of \( f(x) \):
      \[
      \rightarrow 3 - 2x \geq 0 \\
      \frac{3}{2} \geq x \\
      \left( -\infty, \frac{3}{2} \right] 
      \]
   b. Graph \( f(x) \) and label at least three points including the x-intercept:
      \[
      \begin{array}{c|c}
      x & f(x) \\
      \hline
      \frac{3}{2} & 0 \\
      1 & 1 \\
      -\frac{1}{2} & 2 \\
      -3 & 3 \\
      \end{array}
      \]

8. [10] Give the equation of the line in slope-intercept form which is perpendicular to the line \( 8x - 3y = 5 \) and passes through the point \((-2,1)\).
   \[
   \begin{align*}
   8x - 3y &= 5 \\
   -3y &= -8x + 5 \\
   y &= \frac{8}{3}x - \frac{5}{3} \\
   \text{perpendicular to } \frac{8}{3} \text{ is } -\frac{3}{8} = m
   \end{align*}
   \]
   \[
   \begin{align*}
   y - y_1 &= m(x - x_1) \\
   y - 1 &= -\frac{3}{8}(x - (-2)) \\
   y - 1 &= -\frac{3}{8}(x + 2) \\
   y - 1 &= -\frac{3}{8}x - \frac{3}{4} \\
   y &= -\frac{3}{8}x - \frac{3}{4} + 1 \\
   y &= -\frac{3}{8}x + \frac{1}{4}
   \end{align*}
   \]
9. [8] Solve the system of equations:
\[ \begin{align*}
4x - 6y &= 6 \\
-4x + 60y &= 12 \\
\hline
54y &= 18 \\
\hline
y &= \frac{18}{54} = \frac{1}{3}
\end{align*} \]

\[ (2, \frac{1}{3}) \]

10. [6] If \( y \) varies directly with \( x^3 \), and \( y = 14 \) when \( x = 3 \), find \( y \) when \( x = 2 \). Give an exact answer:
\[ \begin{align*}
y &= Kx^3 \\
n &= K(3)^3 \\
n &= K(27) \\
K &= \frac{14}{27}
\end{align*} \]

\[ \begin{align*}
Y &= \frac{14}{27}(2)^3 \\
Y &= \frac{14}{27}(8) = \frac{112}{27}
\end{align*} \]

11. [6] Perform the indicated operations, factor, and reduce the following to simplest form. You may leave your answer in factored form:

a. \( \frac{x^2 - 16}{-3x^2 + x} + \frac{x^2 - 2x - 8}{-x^2 - 2x} \)

\[ \begin{align*}
\frac{(x-4)(x+4)}{x(-3x+1)} + \frac{-x(x+2)}{(x-4)(x+2)} &= \frac{-(x+4)}{(-3x+1)} \\
&\text{or} \quad \frac{(x+4)}{3x-1}
\end{align*} \]
12. [20] Solve the following equations for x. Simplify. Give exact answers:

a. \( \frac{1}{x} - \frac{4}{2x+4} = \frac{7}{8x} \)
\[ \frac{1}{x} - \frac{4}{2(x+2)} = \frac{7}{8x} \]

\[
\left(8x(x+2)\right) \left( \frac{1}{x} - \frac{4}{2(x+2)} \right) = \left(8x(x+2)\right) \left( \frac{7}{8x} \right)
\]

\[
8(x+2)(1) - 4x(4) = 7x(x+2)
\]

\[
8x + 16 - 16x = 7x + 14
\]

\[
-8x + 16 = 7x + 14
\]

\[
2 = 15x \quad \rightarrow \quad x = \frac{2}{15}
\]

b. \( 2 = x + x \at \)

\[
2 = x(1 + a t)
\]

\[
\frac{x}{1 + at} = \frac{2}{1 + at}
\]

c. \( x^2 - 6x = 2 \)

\[
x^2 - 6x - 2 = 0
\]

Does not factor easily

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
a = 1 \\
b = -6 \\
c = -2
\]

\[
x = \frac{6 \pm \sqrt{36 - 4(1)(-2)}}{2(1)} = \frac{6 \pm \sqrt{44}}{2} = \frac{6 \pm 2\sqrt{11}}{2}
\]

\[
= \frac{6 \pm \sqrt{44}}{2} = 3 \pm \sqrt{11}
\]
Section 3: Word problems. Show all work to receive credit. Be sure to use appropriate rounding and include units!

13. [12] Two trucks leave a loading bay at the same time. Truck A is travelling due east and Truck B is travelling due west. Truck B is travelling 11 mph slower than Truck A. After 4.25 hours, the trucks are 455 miles apart. What is the speed of Truck A? Round to two decimal places.

\[ d = r \cdot t \]
\[ d_A = x(4.25) \]
\[ d_B = (x-11)(4.25) \]
\[ d_A + d_B = 455 \text{ mi} \]

\[ x(4.25) + (x-11)(4.25) = 455 \]
\[ 4.25x + 4.25x - 46.75 = 455 \]
\[ 8.5x = 501.75 \]
\[ x = 59.03 \text{ mph} \]

14. [12] How many liters of a 28% acid solution must be mixed with a pure water to produce 5 liters of a 15% acid solution? Round to one decimal place.

\[ 0.28x + 0.75y = 0.15(5) \]
\[ 0.28x + 0 = 0.15(5) \]
\[ 0.28x = 0.75 \]
\[ x = 2.7 \text{ L} \]

15. [10] A flag pole that is 32.0 feet tall is creating a shadow on the ground that is 17.6 feet long. What is the angle of elevation from the end of the shadow to the sun? Round appropriately.

\[ \tan \theta = \frac{32.0}{17.6} \]
\[ \theta = \tan^{-1} \left( \frac{32.0}{17.6} \right) \]
\[ \theta = 61.2^\circ \]
Section 4: Geometry and Trig. Show all work to receive credit. Round appropriately or as indicated in the problem.

16. [10] Find the area of the following:
   a. The area between a square and an inscribed circle where the square has diagonal length of 16.0 in. Round to three significant digits.

   \[ x^2 + x^2 = 16^2 \]
   \[ 2x^2 = 256 \]
   \[ x^2 = 12 \]
   \[ x = 11.314'' \]

   \[ x = \text{diameter of circle} \]
   \[ x = \text{radius of circle} \]

   \[ A_o = \pi \left( \frac{x}{2} \right)^2 = 100.531 \text{ in}^2 \]

   \[ A_p = x^2 = 128 \text{ in}^2 \]

   \[ \frac{A_o - A_p}{A_o} = 0.07 \text{ in}^2 \]

17. [12] Given the three parts of a triangle, find the remaining three parts. You may round off the length of sides to three significant digits and round off angles to the nearest 1/10 of a degree.
   a. \( C = 90.0^\circ, A = 69.0^\circ, c = 12.0 \text{ ft} \)

   \[ \sin A = \frac{a}{c} \]
   \[ \sin (69.0^\circ) = \frac{a}{12.0} \]
   \[ a = 11.2 \text{ ft} \]

   \[ a^2 + b^2 = c^2 \]
   \[ 11.2^2 + b^2 = 12.0^2 \]
   \[ b = 4.30 \text{ ft} \]

   \[ A + B + C = 180^\circ \]
   \[ 69^\circ + B + 90^\circ = 180^\circ \]
   \[ B = 21.0^\circ \]

18. [12] Find the area of a regular hexagon with perimeter of 72.0 feet.

   \[ p = 72.0 \text{ ft} \]
   \[ \text{sum of interior angles} = (6-2)(180^\circ) = 720^\circ \]
   \[ 720^\circ / 6 = 120^\circ = \Theta \]

   \[ \tan(60^\circ) = \frac{h}{p/6} \]
   \[ h = 10.39 \text{ ft} \]

   \[ A_d = \frac{1}{2}(6)(10.39) = 31.18 \]

   \[ A_d \times 12 = A_o = 374 \text{ ft} \]
19. Find the measure of the radius of a sector circle that has a central angle of 34.8° and an arc length of 2.50 inches. Round to three significant digits.

\[ s = \theta r \quad \text{\theta must be in radians} \]

\[
\frac{34.8^\circ}{180^\circ} = 0.607 \text{ rad} 
\]

\[
\frac{s}{0.607} = \frac{2.50}{r} \Rightarrow r = 4.12 \text{ inches} 
\]

20. Given a=81m, B=61.3°, C = 32.8°, find the three remaining parts of an oblique triangle. You may round off the length of sides to the correct number of significant figures and round off angles to the nearest 1/10 of a degree:

Hint: Law of Sines: \( \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \), Law of Cosines: \[
\begin{align*}
\text{Law of Cosines:} & \\
\Rightarrow a^2 &= b^2 + c^2 - 2bc \cos A \\
b^2 &= a^2 + c^2 - 2ac \cos B \\
c^2 &= a^2 + b^2 - 2ab \cos C
\end{align*}
\]

\[
A + B + C = 180^\circ \\
A + 61.3 + 32.8 = 180 \\
A = 85.9^\circ
\]

\[
\frac{b}{\sin B} = \frac{a}{\sin A} \Rightarrow \frac{b}{\sin(61.3^\circ)} = \frac{81}{\sin(85.9^\circ)} \\
\Rightarrow b = 71m
\]

\[
\frac{c}{\sin C} = \frac{a}{\sin A} \Rightarrow \frac{c}{\sin(32.8^\circ)} = \frac{81}{\sin(85.9^\circ)} \\
\Rightarrow c = 44m
\]

21. Sketch a graph of \( f(x) = 2 \sin x \). Be sure to show at least one full period. Label the y-intercept and three other points.