Part I. There are 10 problems in Part I. Little partial credit will be given, so be careful.

[9] 1) Fill in the table: EXACT ANSWERS

<table>
<thead>
<tr>
<th></th>
<th>θ = 30°</th>
<th>θ = 45°</th>
<th>θ = 60°</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin θ</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>( \frac{\sqrt{2}}{2} )</td>
<td>( \frac{\sqrt{3}}{2} )</td>
</tr>
<tr>
<td>cos θ</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>tan θ</td>
<td>( \frac{\sqrt{3}}{3} )</td>
<td>( 1 )</td>
<td>( \sqrt{3} )</td>
</tr>
</tbody>
</table>

2) Perform the indicated operation:

[5] a) Graph \( \sin(x) \) through one period.

[5] b) Graph \( \cos(x) \) through one period.

[5] 3) If \( \theta \) is an acute angle and \( \cos \theta = \frac{1}{5} \), what is \( \sin^2 \theta \)?

[5] 4) Find the dot product of \( \mathbf{u} \) and \( \mathbf{v} \) if \( \mathbf{u} = \langle 2, -3 \rangle \) and \( \mathbf{v} = \langle 1, -2 \rangle \).

\[
\mathbf{u} \cdot \mathbf{v} = 2 \cdot 1 + (-3)(-2) \\
= 2 + 6 \\
= 8
\]
[8] 5) If $\cos(\Theta) = \frac{4}{\sqrt{7}}$ and $\Theta$ is in quadrant 4, find:

a) $\sin(\Theta) = \frac{\sqrt{33}}{7}$

b) $\tan(\Theta) = \frac{\sqrt{33}}{4}$

c) $\sec(\Theta) = \frac{7}{4}$

d) $2\cos(\Theta) - 1 = \frac{5}{7} - \frac{7}{7}$

e) $\Theta = \frac{22}{7}$

[5] 6) What is the equation for the following graph? <5 points>

![Graph]

a) $\frac{x^2}{5} + \frac{y^2}{4} = 1$

b) $\frac{x^2}{25} + \frac{y^2}{16} = 1$

c) $\frac{x^2}{4} + \frac{y^2}{5} = 1$

d) $\frac{x^2}{16} + \frac{y^2}{25} = 1$

e) Not listed

[5] 7) Which of the following are coterminal to an angle of 70 degrees?

a) 20 degrees  b) 110 degrees  c) 430 degrees  d) 290 degrees  e) Not a, b, c or d

70 + 360 = 430

[5] 8) Write $\frac{2\pi}{5}$ in degrees. $

\left(\frac{2\pi}{5}\right) \left(\frac{180}{\pi}\right) \frac{360}{5} = 72^\circ$
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[5] 9) What is phase shift for the graph: \( y = -3 \sin(2x - 3) + 5 \)?
   a) 5  b) 3/2  c) 3  d) -3/2  e) not a, b, c or d

[7] 10) Find the length of side b and \( \tan(\theta) \).

\[
\left( \frac{5}{2} \right)^2 + b^2 = 3 + \sqrt{3} \\
\frac{25}{4} - 3 = b^2 \\
\frac{25}{4} - \frac{13}{4} \leq \frac{13}{4} = b^2 \\
b = \frac{\sqrt{13}}{2}
\]

\[b_0 \theta = \frac{\sqrt{13}}{2} = \frac{\sqrt{3} \cdot \sqrt{3} \cdot \sqrt{3}}{6} = \frac{\sqrt{3}}{6}
\]

**Part II. Show all your work. Each problem is worth 6 points.**

[6] 11) Given \( u = (-2, 4) \) and vector \( v = (-1, -1) \) what is \(|4u - v| = ?

\[
4u = (-8, 16)
\]

\[
|(-8, 16) - (-1, -1) |
\]

\[
|(-7, 17)| = \sqrt{(-7)^2 + 17^2} = \sqrt{334}
\]

[6] 12) Suppose that P is a point on a circle with a radius of 10 inches and the ray OP is rotating with angular speed 100 degrees per second.

a) Find the speed in radians per second.

\[
\left(10 \, 0^\circ \right) \left( \frac{\pi}{180^\circ} \right) = \frac{10}{18} = \frac{5\pi}{9}
\]

b) Find the distance travelled by P along the arc after 1 second.

\[
s = \frac{5\pi}{9} \cdot 10
\]

\[
s = \left(\frac{5\pi}{9}\right)(10) = \frac{50\pi}{9}
\]
[6] 13) Given \( \sin x = 0.2 \), state the solution set on \([0,360^\circ)\). Approximate to nearest degree. Show all work clearly.

\[
\begin{align*}
\arcsin(0.2) &\approx 12^\circ \\
&\approx 168^\circ
\end{align*}
\]

[6] 14) If the rectangular coordinates of a point are \((4, -4)\), what are its polar coordinates \((r, \theta)\) given the following?

a) \( r > 0, \ 0 \leq \theta < 2\pi \)

\[
\left(4\sqrt{2}, \ \frac{7\pi}{4}\right)
\]

b) \( r < 0, \ 0 \leq \theta < 2\pi \)

\[
\left(-4\sqrt{2}, \ \frac{3\pi}{4}\right)
\]

[6] 15) Write the trigonometric expression as an algebraic expression in terms of \( u \) (\( u > 0 \))

\[
\csc(\tan^{-1} u)
\]

[6] 16) Perform the indicated operation:

\[
\begin{align*}
a) \ &\text{Factor:} \ 2\sin^2 x - 4\cos x\sin x \\
\ &\quad = 2\sin x(\sin x - 2\cos x) \\
b) \ &\text{Simplify:} \ \frac{\cos^2 x}{1 - \sin^2 x} \\
\ &\quad = \frac{\cos^2 x}{\cos^2 x}
\end{align*}
\]
Part III. Partial credit will be given here. Show all your work.

[12] 17) Write an equation for each.

b) Answer $-5 \sin \left( \frac{1}{2} x \right)$

b) Answer $\cos(2x) - 1$

Graph $f(x) = \sin^{-1} x$.
Label axes with at least 2 ticks each.
18) [4] a) Write in rectangular form: (2, 120°) [4]b) convert to polar equation: \( x^2 + y^2 = 36 \)

\[ (-1, \sqrt{3}) \]

\[ r = \sqrt{36} \]

\[ r = 6 \]

[8] 19) Verify (prove): \( \frac{\tan x - \cot x}{\sin x \cos x} = \sec^2 x - \csc^2 x \)

Include all steps and explanations

\[
\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} = \frac{\sin^2 x - \cos^2 x}{\sin x \cos x}
\]

\[
\frac{\sin^2 x - \cos^2 x}{\sin x \cos x} = \frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} = \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} = \sec^2 x - \csc^2 x
\]

[7] 20) a) Graph the polar equation \( r = 4 \sin \theta \) on the axes below.
21) Solve the following.

[5] a) Find all solutions to \(4 \cos^2 x - 3 = 0\). Express in terms of degrees.

\[4 \cos^2 x = \frac{3}{4}\]

\[\cos x = \pm \frac{\sqrt{3}}{2}\]

\[x = \pm \frac{\pi}{6}, \frac{5\pi}{6}\]

Note: All solutions are the same as general solutions.

[5] b) \(2 \sin^2 x = 1 - \sin x\) on \([0, 2\pi)\)

\[2 \sin^2 x - \sin x - 1 = 0\]

\[(2 \sin x - 1)(\sin x + 1) = 0\]

\[2 \sin x = 1 \quad \sin x = -1\]

\[
\begin{align*}
\sin x = \frac{1}{2} & \quad x = \frac{\pi}{6}, \frac{5\pi}{6} \\
\sin x = -1 & \quad x = \pi
\end{align*}
\]

[6] 22) Solve the RIGHT triangle, ABC: \(a = 12\) inches, \(b = 9\) inches. Round to two decimal places. Use degrees and put units on answers.

\(c = 15\) inches

\(C = 90\) degrees

\(A = 53.13^\circ\)

\(B = 36.87^\circ\)

[6] 23) Change \(2y^2 - 12x + 3x^2 = 6\) into standard form. Identify the graph as an ellipse, circle, parabola. Then graph (include center and foci if necessary).

\[\frac{(x - 2)^2}{6} + \frac{y^2}{9} = 1\]

\[c = 3\]

\[c = \sqrt{3}\]
PART IV. Here are 6 problems. Do any 4, but only 4. Each is worth 10 points. Be sure to check the box for each problem to be graded.

24) Find cube roots of $1 + i$. Leave answers in trig form.

\[ (\sqrt{2} \cos 45^\circ + i \sin 45^\circ)^{\frac{1}{3}} \]

\[ 2) \quad \sqrt[3]{2} \left( \cos 15^\circ + i \sin 15^\circ \right) \]

\[ 2_2) \quad \sqrt[3]{3} \left( \cos 135^\circ + i \sin 135^\circ \right) \]

\[ 2_3) \quad \sqrt[3]{3} \left( \cos 255^\circ + i \sin 255^\circ \right) \]

25) Graph the following. Indicate and label all critical information.

\[ \frac{(y-3)^2}{9} + \frac{(x+2)^2}{4} = 1 \]

- Center: 
- Vertices: 
- Foci: 

26) An airplane is flying at a height of 2 miles above the ground. The distance along the ground from the airplane to the airport is 5 miles. What is the angle of depression from the airplane to the airport? Round to 1 decimal place.
27) Points A & B are on opposite sides of a lunar crater. Point C is 50 m from point A. The measure of angle BAC is 112 degrees and the measure of angle ABC is 38 degrees. What is the width of the crater?

\[
\sin 38^\circ = \frac{x}{50} \\
\sin 112^\circ = \frac{50}{x}
\]

\[x = 40.6 \text{ m}
\]

28) Prove the following identity: \( \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \)

\[
\tan(A + B) = \frac{\sin(A + B)}{\cos(A + B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}
\]

\[
= \frac{\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B - \sin A \sin B}{\cos A \cos B}} = \frac{\tan A + \tan B}{1 - \tan A \tan B}
\]

29) Two forces of 28 N and 35 N act on objects at right angles.

a) Find the magnitude of the resultant vector

\[\gamma = \sqrt{35^2 + 28^2} = 44.8 \text{ N}
\]

b) Find the angle the resultant vector makes with the smaller force.

\[\alpha = \tan^{-1}\left(\frac{35}{28}\right) = 51.3^\circ
\]