Langenhop Lecture and SIU Pure Mathematics Conference

Department of Mathematics
Southern Illinois University Carbondale
Carbondale, Illinois, USA

May 14–15, 2019

Logic

Rachel Alvir (University of Notre Dame)

Lexi Block Groman (University of Illinois-Urbana-Champaign)

Tyler Brown (Iowa State University)

Wesley Calvert (Southern Illinois University Carbondale)

Hunter Chase (University of Illinois, Chicago)

Simon Cho (University of Michigan)

Gabriel Conant (University of Notre Dame)

Michael Cotton (University of North Texas)

Rumen Dimitrov (Western Illinois University)

Damir Dzhafarov (University of Connecticut)

James Freitag (University of Illinois, Chicago)

Valentina Harizanov (George Washington University)

Nadja Hempel (UCLA)

Philipp Hieronymi (University of Illinois, Urbana-Champaign)

Oscar Levin (University of Northern Colorado)

Carl Mummert (Marshall)

Ivan Ongay Valverde (University of Wisconsin)

Wim Ruitenburg (Marquette University)

Dodamgodage Gihanee Senadheera (SIUC)

Henry Towsner (University of Pennsylvania)

Rose Weisshaar (University of Pennsylvania)

Tuesday, May 14, 2019

Kwangho Choiy(Chair)

Algebra & Number Theory	**Neckers 240**
David Leep	8:50-9:35
Lisa Kaylor	9:40-10:00
Ralf Schmidt	10:05-10:50
Rodney L. Keaton	10:55-11:15
Dylon Chow	11:20-11:40

Dubravka Ban(Chair)

Algebra & Number Theory	**Neckers 240**
Shuichiro Takeda	1:00-1:45
Daniel Shankman	1:50-2:10
Wai Kiu Chan	2:15-3:00
Lenny Fukshansky	3:05-3:25
Ravindra Girivaru	3:30-3:50

Jerzy Kocik(Chair)

Combinatorics & Geometry	**Nckers 440**
Shoichi Fujimori	9:00-9:45
Charles Delman	9:50-10:35
Louis Kauffman	10:40-11:25

Jerzy Kocik(Chair)

Combinatorics & Geometry	**Nckers 440**
Indu Satija	1:00-1:45
Zbigniew Oziewicz	1:50-2:35
Bill Page	2:40-3:10
Robert Owczarek	3:15-3:45

Wesley Calvert(Chair)

Logic	**Nckers 440**
Henry Townsner	9:00-9:20
Gabriel Conant	9:30-9:50
Alexi Block Gorma	10:00-10:20
Carl Mummert	10:30-10:50
Oscar Levin	11:00-11:20

Wesley Calvert(Chair)

Logic	**Nckers 440**
James Freitag	1:00-1:25
Simon Cho	1:30-1:50
Rose Weisshaar	2:00-2:20
Ivan Ongay Valverde	2:30-2:50
Nadja Hempel	3:00-3:20

(1) Registration on Monday begins at 8:00 at the atrium of Neckers, followed by a welcome session 8:30-8:45 in Neckers Room 240.
(2) The Langenhop Lecture is in Guyon Auditorium of Morris Library.
(3) The lunch is 11:30-1:00 pm.

Tuesday, May 15, 2019

Andrew Earnest(Chair)

Algebra & Number Theory	**Neckers 240**
Anna Haensch	8:30-8:50
Lakshika Gunawardana	8:55-9:15
Yeongseong Jo	9:20-9:40
Chathurika Athapattu	9:45-10:05
Bogdan Petrenko	10:10-10:55
Benjamin Hutz	11:00-11:20
Jireh Loreaux	11:25-11:45

Kwangho Choy(Chair)

Algebra & Number Theory	**Neckers 240**
Haohao Wang	1:30-2:00
Henri Shurz	2:00-2:30

Jerzy Kocik(Chair)

Combinatorics & Geometry	**Nckers 440**
Mark Ellingham	9:00-10:00
Dinush Panditharathna	10:00-10:30
Coffee Break	10:30-11:00
John McSorley	11:00-11:30

Jerzy Kocik(Chair)

Combinatorics & Geometry	**Nckers 440**
Indu Satija	1:00-1:45
Zbigniew Oziewicz	1:50-2:35
Bill Page	2:40-3:10
Robert Owczarek	3:15-3:45

Wesley Calvert(Chair)

Logic	**Nckers 156**
Philipp Hieronymi	8:30-8:50
Rachael Alvir	9:30-9:50
Michael Cotton	10:00-10:20
Wim Ruitenburg	10:30-10:50
Tyler Brown	11:00-11:20
Valentina Harizanov	

Wesley Calvert(Chair)

Logic	**Nckers 156**
Damir Dzhafarov	1:00-1:20
Hunter Chase	1:30-1:50
Rumen Dimitrov	2:00-2:20
Gihanee Senadheera	2:30-2:50

Logic Abstracts

Scott Sentences of Scattered Linear Orders

Rachael Alvir*
Department of Mathematics
University of Notre Dame
Dino Rossegger
Institute of Discrete Mathematics and Geometry
Vienna University of Technology

The logic $L_{\omega_1\omega}$ is obtained by closing finitary first-order logic under countable disjunction and conjunction. There is a kind of normal form for such sentences. For any structure there is a sentence of $L_{\omega_1\omega}$, known as its *Scott sentence*, which describes A up to isomorphism among countable structures. Given a countable scattered linear order L of Hausdorff rank $\alpha < \omega_1$ we show that it has a $\inf 2\alpha + 1$ Scott sentence. From Ash's calculation of the back and forth relations for all countable well-orders, we obtain that this upper bound is tight, i.e., for every $\alpha < \omega_1$ there is a linear order whose optimal Scott sentence has this complexity. We further show that for all countable α the class of Hausdorff rank α linear orders is $\mathbf{\Sigma}_{2\alpha+2}$ complete.

Characterizing Companionability for Expansions of an O-minimal Group by a Generic Subgroup

Alexi Block Gorman
Department of Mathematics
University of Illinois Urbana-Champaign

In this talk, we present a characterization for when the theory of an o-minimal expansion of an ordered group augmented by a predicate for a dense and codense subgroup has a model companion. The characterization is purely geometric in nature; indeed, such a pair $(\mathcal{R}, \mathcal{G})$ has a model companion if and only if there is no infinite definable family of distinct local endomorphisms of $\mathcal{G}$ in $\mathcal{R}$. In the case that $\mathcal{R}$ expands an o-minimal field and $\mathcal{G} \leq \mathcal{R}_{>0}$, the characterization easily reduces to $(\mathcal{R}, \mathcal{G})$ companionable if and only if no restriction of $x \mapsto 2^x$ to an interval is definable in $\mathcal{R}$.

This work is motivated by recent characterizations of companionability for broader classes of geometric structures with generic reducts, such as those found in "Generic Expansions by a Reduct" by Christian D'Elbée and "Interpolative Fusions" by Alex Kruckman, Minh Chieu Tran, and Erik Walsberg. Indeed, we will discuss the satisfying way our characterization (for expansions of o-minimal groups with a predicate for a generic subgroup) fits perfectly into the dichotomies drawn in the aforementioned works, and illustrates the principles behind the dividing lines that the above authors have identified.

Computing Unity in $C[0,1]$

Tyler A. Brown
Department of Mathematics
Iowa State University

In 2014, A.G. Melnikov and K.M. Ng demonstrated that $C[0,1]$ as a Banach space is not computably categorical by exhibiting a computable presentation in which the unit function **1** is not computable. In this talk we determine a Turing degree sufficient for computing the unit function **1** in any given computable presentation of $C[0,1]$. Time permitting, we will also exhibit computability results for several other operations within the context of this ubiquitous space.

Model-theoretic techniques in query learning

Hunter Chase
Department of Mathematics, Statistics, and Computer Science
University of Illinois at Chicago

Several notions of complexity of set systems correspond both with model-theoretic dividing lines and notions of machine learning. NIP formulas correspond with PAC-learning by way of VC-dimension, and stable formulas correspond with online learning by way of Littlestone dimension, also known as Shelah 2-rank. We extend this correspondence to equivalence query learning. The relevant notions of complexity are the consistency dimension and strong consistency dimension, which roughly correspond with NFCP formulas. By considering these measures alongside Littlestone dimension, we obtain new bounds for several variants of equivalence query learning and extend known results to the infinite case.

Categorical semantics of metric structure

Simon Cho
Department of Mathematics
University of Michigan

Categorical semantics is the study of how different categories may support a given logical structure; its pursuit has produced deep results e.g. describing conditions under which various fragments of classical logic may be interpreted into a given category. Motivated by the development of continuous logic - which generalizes classical logic by extending the usual binary truth values to the real interval [0, 1] and replaces sets with metric spaces - we investigate the categorical semantics of some aspects of this logic. We recall basic notions of categorical semantics, and use the category of metric spaces as a template to develop a "continuous semantics" which then allows us to ask if objects of general categories possess metric space-like properties. We will show that the category of metric spaces has a "continuous subobject classifier which is a natural generalization of the usual subob-ject classifier; time permitting, we may exhibit the category of presheaves of metric spaces as a "category of metric spaces" and even show that it has a continuous subobject classifier.

Stability in a group

Gabriel Conant
Department of Mathematics
University of Notre Dame

We develop local stable group theory directly from topological dynamics, and extend the main results in this subject to the setting of stability "in a model". Specifically, given a group G, we analyze the structure of sets $A \subseteq G$ such that the bipartite relation $xy \in A$ omits infinite half-graphs. Our proofs rely on the characterization of stability via Grothendieck's "double-limit" theorem (as shown by Ben Yaacov), and the work of Ellis and Nerurkar on weakly almost periodic G-flows.

Abelian group actions and hypersmooth equivalence relations

Michael Cotton
Department of Mathematics
University of North Texas

We show that any Borel action on a standard Borel space of a group which is isomorphic to the sum of a countable abelian group with a countable sum of lines and circles induces an orbit equivalence relation which is hypersmooth. We also show that any Borel action of a second countable locally compact abelian group on a standard Borel space induces an orbit equivalence relation which is essentially hyperfinite, generalizing a result of Gao and Jackson for the countable abelian groups.

Cohesive Powers of Linear Orders

Rumen Dimitrov
Department of Mathematics and Philosophy
Western Illinois University

Abstract: Cohesive powers of computable fields were used in the study of structure and automorphisms of the lattice $L^*(V_\infty)$. In this talk I will survey known results about cohesive powers and will show that different computable presentations of a computable structure may have nonisomorphic cohesive powers.

Some new results concerning the SRT^2_2 vs. COH problem

Damir D. Dzhafarov
Department of Mathematics
University of Connecticut

The SRT^2_2 vs. COH problem is a central problem in computable combinatorics and reverse mathematics, asking whether every Turing ideal that satisfies the principle SRT2 also satisfies the principle COH. We present some results towards further developing some of the main techniques involved in attacking this problem. We study several principles related to each of SRT^2_2 and COH, and prove results that highlight the limits of our current understanding, but also point to new directions ripe for further exploration. This is joint work with Peter Cholak, Denis Hirschfeldt, and Ludovic Patey.

Model theory, functional transcendence & unlikely intersections

James Freitag
Department of Mathematics, Statistics, & Computer Science
University of Illinois at Chicago

In this talk, we will give a general introduction to a number of problems of unlikely intersections, a class of number theoretic problems. Well-known examples include Manin-Mumford and André-Oort type problems. We will also explain the relationship between problems of unlikely intersections and certain classical functional transcendence conjectures. Model theory provides part of the link as well as some essential tools which have been used in the recent resolution of a number of problems in both areas.

Chensive powers, linear orders, and definability(Part II)

Valentina Harizanov
Department of Mathematics
George Washington University.

Cohesive products of computable structures can be viewed as effective ultraproducts over cohesive sets. A set is cohesive if it is infinite and indecomposable with respect to computably enumerable sets. The elements of cohesive products are equivalence classes of partial computable functions. We investigate the isomorphism types of cohesive power for computable linear order. While a cohesive power satisfies the same $\exists\forall\exists$-sentences as the structure, and preserves the satisfiability of all $\exists\forall\exists$-sentences, we provide examples of computable linear order and $\exists\forall\exists$-sentences they satisfy, which may not be satisfied in cohesive powers. This work is join with Dimitrow, Morozov. Shafer, A. Soskova, and Vatey.

n-dependent groups and fields

Nadja Hempel
Department of Mathematics
UCLA.

NIP theories are the first class of the hierarchy of n-dependent structures. The random n-hypergraph is the canonical object which is n-dependent but not (n-1)-dependent. Thus the hierarchy is strict. But one might ask if there are any algebraic objects (groups, rings, fields) which are strictly n-dependent for every n?

We will start by introducing the n-dependent hierarchy and present all known results on n-dependent groups and fields. These were (more or less) inspired by the above question.

Undecidability of $(\mathbb{N}, <, +, 2^{\mathbb{N}}, 3^{\mathbb{N}})$

Philipp Hieronymi
Department of Mathematics
University of Illinois

In 1985 Lou van den Dries asked whether or not the theory of $(\mathbb{N}, <, +, 2^{\mathbb{N}}, 3^{\mathbb{N}})$ is decidable? In this talk I will show that this theory is undecidable. This is joint work with Christian Schulz.

Reverse Mathematics of Maximal and Perfect Matchings

Oscar Levin
School of Mathematical Sciences
University of Northern Colorado

A *matching* of a graph is a subset of its edges no two of which are incident to any single vertex, i.e., a vertex is *matched* to at most one other vertex by an edge in the matching. Matchings for *bipartite* graphs are well understood classically, and these classical theorems have been studied extensively in reverse mathematics. The situation for graphs in general is more complicated. Classically, in a 1977 paper, K. Steffens proved that all graphs have maximal matchings, even when we measure maximality with respect to the set of vertices matched. Further, an infinite graph has a perfect matching (i.e., ever vertex is matched) precisely when it is always possible to increase the support of any imperfect matching. In this talk, we will consider the complexity of these "obvious" theorems. We will see that the situation for locally finite graphs is as elegant as one would expect, but that allowing for infinite degree vertices makes the analysis surprisingly difficult.

Reverse mathematics and excluded middle

Carl Mummert
Department of Mathematics
Marshall University

I will discuss work with Jeffry Hirst on relationships between reverse mathematics, the law of the excluded middle, and Weihrauch reducibility. We are particularly interested in finding methods using classical computability theory to show that particular theorems or reductions are not provable without the use of excluded middle. We are also interested in formalizing Weihrauch reductions, which are ordinarily defined without mention of any verification in a formal system.

Proofs without excluded middle and Weihrauch reductions each suggest notions of uniformity – methods of obtaining a problem output from a problem input in the same way for all inputs. We study these notions of uniformity using systems of constructive second order arithmetic. The use of constructive systems and formalized Weihrauch reducibility also gives a new way to characterize when one theorem can be proved with a single use of a second theorem.

The case of the surviving degrees

Ivan Ongay Valverde* & Noah Schweber
Department of Mathematics
University of Wisconsin-Madison

In recent years, Rupprecht show an analogy between cardinal characteristics of the continuum (Set Theory) and highness notions (Computability Theory). Using his machinery, it is possible to translate one into the other changing inequalities for implications. Although, most of the results in the computability side are analogues from the set theory one, there exist some cases where they differ. This scheme has inspire a number of papers working over the, now called, Effective Cichoń's Diagram.

In this talk, we will quickly introduce the translation scheme and we will show work done over the localization numbers (introduce by Newelski and Rosnlanowski) and their analogues, the surviving degrees.

Intuitionistic Logic versus Constructive Logic

Wim Ruitenburg
Department of Mathematics, Statistics, and Computer Science
Marquette University, Milwaukee, WI

We develop a new predicate logic for constructive mathematics simultaneously with a matching new proof interpretation.
We give a brief 'historical' overview of the major schools of constructive mathematics. Many quotes from the literature imply that the insights that result in our re-interpretation, have a significant precursor in earlier insights.

Existence of PAC Incomparable Degrees

Wesley Calvert & Gihanee Senadheerat
Department of Mathematics
Southern Illinois University Carbondale

Probably Approximately Correct (PAC) Learning is one of the models C_0 used in machine learning. This model was proposed by Valiant in 1984. The work related to this abstract was inspired by the Post's problC_0em. Post classified computably enumerable (c.e.) sets and their degrees and was interested in fnding more than two c.e. degrees. This was known as the Post's problem.In 1957 Friedberg and Muchnik independently showed that is possible.

In PAC learning model, there are concept classes which are learnable and also there are concept classes which are hard to learn. Later mathematicians were able to postulate the notion of PAC reducibility. That is, if a concept class C_0 is PAC learnable through an algorithm then the concept class C_1 reduces to the concept class C_0 means C_1 can be leaC_0rned through the existing algorithm for C_0 . The term PAC degree means degree of unsolvability of a PAC concept class. It is natural to ask the question whether there are incomparable PAC degrees. In order to prove that there are incomparable PAC degrees we use the C_0 method known as priority construction, which is used by Friedberg C_0 Muchnik in their work. We construct two concept classes C_0 and C_1 such that C_0 is not reducible to C_1 and C_1 is not reducible to C_0.

Why (and when) is there only one way to random?

Henry Towsner
Department of Mathematics
University of Pennsylvania

We can think of the eigenvalues of a matrix as describing how structured it is: a matrix is "more structured" if it has a few large eigenvalues, and "close to random" if it has a large number of small eigenvalues.

With *tensors*, the picture becomes more complicated: we can identify several different kinds of "eigenvalues" and "eigenvectors", and identify tensors which are "random" for one kind of eigenvector and "non-random" for a different kind of eigenvector. Each notion of eigenvector captures one kind of "structure" a tensor might have.

Mathias generics over a countable Turing ideal

Henry Towsner
Department of Mathematics
University of Pennsylvania
Dan Turetsky
Department of Mathematics
Victoria University of Wellington
Rose Weisshaar *
Department of Mathematics
University of Pennsylvania

We consider effective Mathias forcing over a countable Turing ideal. In particular, we show that a Mathias generic constructed with conditions from a countable ideal $\mathcal{I}$ need not compute a Mathias generic over every smaller ideal $\mathcal{J} \subseteq \mathcal{I}$, answering a question posed by Cholak, Dzhafarov, and Soskova.