Langenhop Lecture and SIU Pure Mathematics Conference

Department of Mathematics
Southern Illinois University Carbondale
Carbondale, Illinois, USA

May 14–15, 2019

Algebra and Number Theory

Chathurika Athapattu(Southern Illinois University Carbondale)

Wai Kiu Chan (Wesleyan University)

Dylon Chow (University of Illinois at Chicago)

Lenny Fukshansky (Claremont McKenna College)

Larry Gerstein (UC Santa Barbara)

Ravindra Girivaru (University of Missouri - St. Louis)

Lakshika Gunawardana (Southern Illinois University Carbondale)

Anna Haensck (Duquesne University)

Benjamin Hutz (Saint Louis University)

Yeongseong Jo (University of Iowa)

Lisa Kaylor (Wesleyan University)

Rodney L. Keaton (East Tennessee State University)

David Leep (University of Kentucky)

Jireh Loreaux (Southern Illinois University Edwardville)

Bogdan Petrenko (Eastern Illinois University)

Ralf Schmidt (University of Oklahoma)

Daniel Shankman (Purdue University)

Shuichiro Takeda (University of Missouri-Columbia)

Tuesday, May 14, 2019

Kwangho Choiy(Chair)

Algebra & Number Theory	**Neckers 240**
David Leep	8:50-9:35
Lisa Kaylor	9:40-10:00
Ralf Schmidt	10:05-10:50
Rodney L. Keaton	10:55-11:15
Dylon Chow	11:20-11:40

Dubravka Ban(Chair)

Algebra & Number Theory	**Neckers 240**
Shuichiro Takeda	1:00-1:45
Daniel Shankman	1:50-2:10
Wai Kiu Chan	2:15-3:00
Lenny Fukshansky	3:05-3:25
Ravindra Girivaru	3:30-3:50

Jerzy Kocik(Chair)

Combinatorics & Geometry	**Nckers 440**
Shoichi Fujimori	9:00-9:45
Charles Delman	9:50-10:35
Louis Kauffman	10:40-11:25

Jerzy Kocik(Chair)

Combinatorics & Geometry	**Nckers 440**
Indu Satija	1:00-1:45
Zbigniew Oziewicz	1:50-2:35
Bill Page	2:40-3:10
Robert Owczarek	3:15-3:45

Wesley Calvert(Chair)

Logic	**Nckers 440**
Henry Townsner	9:00-9:20
Gabriel Conant	9:30-9:50
Alexi Block Gorma	10:00-10:20
Carl Mummert	10:30-10:50
Oscar Levin	11:00-11:20

Wesley Calvert(Chair)

Logic	**Nckers 440**
James Freitag	1:00-1:25
Simon Cho	1:30-1:50
Rose Weisshaar	2:00-2:20
Ivan Ongay Valverde	2:30-2:50
Nadja Hempel	3:00-3:20

(1) Registration on Monday begins at 8:00 at the atrium of Neckers, followed by a welcome session 8:30-8:45 in Neckers Room 240.
(2) The Langenhop Lecture is in Guyon Auditorium of Morris Library.
(3) The lunch is 11:30-1:00 pm.

Tuesday, May 15, 2019

Andrew Earnest(Chair)

Algebra & Number Theory	**Neckers 240**
Anna Haensch	8:30-8:50
Lakshika Gunawardana	8:55-9:15
Yeongseong Jo	9:20-9:40
Chathurika Athapattu	9:45-10:05
Bogdan Petrenko	10:10-10:55
Benjamin Hutz	11:00-11:20
Jireh Loreaux	11:25-11:45

Kwangho Choy(Chair)

Algebra & Number Theory	**Neckers 240**
Haohao Wang	1:30-2:00
Henri Shurz	2:00-2:30

Jerzy Kocik(Chair)

Combinatorics & Geometry	**Nckers 440**
Mark Ellingham	9:00-10:00
Dinush Panditharathna	10:00-10:30
Coffee Break	10:30-11:00
John McSorley	11:00-11:30

Jerzy Kocik(Chair)

Combinatorics & Geometry	**Nckers 440**
Indu Satija	1:00-1:45
Zbigniew Oziewicz	1:50-2:35
Bill Page	2:40-3:10
Robert Owczarek	3:15-3:45

Wesley Calvert(Chair)

Logic	**Nckers 156**
Philipp Hieronymi	8:30-8:50
Rachael Alvir	9:30-9:50
Michael Cotton	10:00-10:20
Wim Ruitenburg	10:30-10:50
Tyler Brown	11:00-11:20
Valentina Harizanov	

Wesley Calvert(Chair)

Logic	**Nckers 156**
Damir Dzhafarov	1:00-1:20
Hunter Chase	1:30-1:50
Rumen Dimitrov	2:00-2:20
Gihanee Senadheera	2:30-2:50

Algebra and Number Theory Abstracts

Parabolic induction for p-adic Banach space representations

Chathurika Athapattu & Dubravka Ban
Department of Mathematics
Southern Illinois University Carbondale

We study the parabolic induction for p-adic Banach space representations of p-adic groups. Using the Schneider-Teitelbaum duality, we consider the corresponding Iwasawa modules. We express the dual of parabolically induced representations in terms of tensor products.

Abstract for LLMC on Algebra & Number Theory

Wai Kiu Chan
Department of Mathematics and Computer Science
Wesleyan University

Let $Q(x_1, \ldots, x_n)$ be an isotropic quadratic form over a number field and t be a nonzero element in that number field. In this talk I will discuss a recent joint work with L. Fukshanksy on the height bounds of the rational or integral solutions to the equation $Q(x_1, \ldots, x_n) = t$ which are outside of some algebraic sets.

Integral Points on the Wonderful Compactification by Height

Dylon Chow
Department of Mathematics, Statistics, & Computer Science
University of Illinois Chicago

A conjecture of Batyrev and Manin predicts the distribution of rational points of bounded height on a wide class of projective varieties. An analogous conjecture predicts the distribution of integral points. The case of the wonderful compactification was proven in two ways, one using harmonic analysis on adele groups, and the other using techniques from dynamics. In this talk I will give an overview of these conjectures and discuss the input from automorphic forms to prove special cases.

Lattices from group frames and vertex transitive graphs

Lenny Fukshansky
Claremont McKenna College

Tight frames in Euclidean spaces are widely used convenient generalizations of orthonormal bases. A particularly nice class of such frames is generated as orbits under irreducible actions of finite groups of orthogonal matrices: these are called irreducible group frames. Integer spans of rational irreducible group frames form Euclidean lattices with some very nice geometric properties, called strongly eutactic lattices. We discuss this construction, focusing on an especially interesting infinite family in arbitrarily large dimensions, which comes from vertex transitive graphs. We demonstrate several examples of such lattices from graphs that exhibit some rather fascinating properties. This is joint work with D. Needell, J. Park and J. Xin.

Matrix representations of polynomials

Ravindra Girivaru
Department of Mathematics and CS
University of Missouri – St. Louis

A rather long standing question, dating back to at least Dickson's work in 1921, is the following:

> given a homogeneous polynomial of degree d in n variables, is it possible to obtain some power of it as the determinant of a matrix whose entries are linear homogeneous polynomials in the same variables.

This question is intimately connected with the geometry of hypersurfaces in projective space. We will give a brief survey of some of the work done in this area.

A Primitive Counterpart of the Fifteen Theorem

A. G. Earnest and B. L. K. Gunawardana
Department of Mathematics
Southern Illinois University Carbondale

In 1993, J. H. Conway and W. Schneeberger presented the Fifteen Theorem for classically integral quadratic forms. Later in 2000, M. Bhargava provided a refinement of the Fifteen Theorem and showed that there are exactly 204 positive definite classically integral quaternary quadratic forms, up to equivalence, which are universal . We try to determine which of the forms in 204 list are primitively universal, and try to determine whether there exists a finite set S of integers such that every positive definite integral quadratic form that primitively represents the integers in S, primitively represents all positive integers. In this talk, we present a conjecture which could be a primitive counterpart to the Fifteen Theorem.

Quadratic forms and the representation problem

Anna Haensch
Department of Mathematics and Computer Science
Duquesne University

Given a polynomial $f(x)$ of several variables with rational coefficients and an integer n, we say that f represents n if the equation $f(x) = n$ is solvable in the integers. One might ask, is it possible to effectively determine the set of integers represented by f? This so-called representation problem for quadratic polynomials is one of the classical problems in number theory. The negative answer to Hilbert's 10th problem tells us that in general, there is no finite algorithm to decide whether a solution exists. However work of Siegel in the 1970's shows that in the case of quadratic polynomials the representation problem is tractable. Algebraically, we can view the representation problem using the language of integral quadratic lattices and lattice cosets, but this method stops short of realizing a full local-global principle. Melding this with the analytic approach of studying the coefficients of theta series we can reach some satisfying solutions to the representation problem for certain families of polynomials. In this talk we will illuminate several important connections between the algebraic and analytic theory of quadratic lattice and finish with a conjecture involving a Siegel-Well type formula for representations by inhomogeneous quadratic polynomials.

Smallest representatives of $SL(2, ZZ)$-orbits of binary forms and endomorphisms of PP^1

Benjamin Hutz
Department of Mathematics and Statistics
Saint Louis University
Michael Stoll
Mathematisches Institut
Universitat Bayreuth

We develop an algorithm that determines, for a given squarefree binary form F with real coeffcients, a smallest representative of its orbit under $SL(2, ZZ)$, either with respect to the Euclidean norm of the coeffcient vector or with respect to the global height. This is based on earlier work of Cremona and Stoll. We then generalize our approach so that it also applies to the problem of finding a representative of smallest height in the $PGL(2, ZZ)$ conjugacy class of an endomorphism of the projective line. Having a small model of such an endomorphism is useful for various computations.

Rankin-Selberg L-functions via Good Sections

Yeongseong Jo
Department of Mathematics
The University of Iowa

In 1990's Bump and Ginzburg establish the integral representation yielding symmetric square L-functions for $GL(n)$ and the twisted version is recently constructed by Takeda. Unfortunately the local functional equation involves intertwining operator opposed to Fourier transform appearing in the well-known Rankin-Selberg integrals for $GL(n) \times GL(n)$ by Jacquet, Piatetski-Shapiro, and Shalika. In this talk, we investigate the modified integrals to incorporate intertwining operator at finite ramifed places. It turns out that "Good Section" introduced by Piatetski-Shapiro and Rallis posses desired properties to build the functional equation contrary to holomorphic section. If time permits, I describe how this framework is relevant to computing local symmetric square L-functions for $GL(n)$.

Quaternary Even Positive Definite Lattices of Discriminant $4p$

Lisa Kaylor
Department of Mathematics & Computer Science
Wesleyan University

In this talk, we consider positive definite even quaternary integral lattices and their theta series. For such lattices with discriminant 389 and minimum 2, Kitaoka showed that there is a linear dependence relation among the theta series corresponding to the classes of these lattices. However, Hsia and Hung showed that the degree 2 theta series corresponding to the classes of positive definite even quaternary integral lattices of discriminant p a prime congruent to 1 mod 4 with minimum 2 are linearly independent. We consider those lattices with discriminant $4p$ where $p > 13$ is a prime congruent to 3 mod 4. There are two genera of lattices in this case, which are considered separately. We follow the strategy of Hsia and Hung to show that the degree 2 theta series of the classes with nontrivial orthogonal group are linearly independent within each genus.

How often is the order of a point on an elliptic curve coprime to q?

Rodney Keaton
Department of Mathematics and Statistics
East Tennessee State University

Let E be an elliptic curve over the rationals and fix a prime q. We want to investigate when the reduction of E modulo a prime p has a point of order coprime to q. In 2010, Jones and Rouse computed the density of such primes p among all primes. In this talk, we will be computing the the error term involved in this density calculation.

Quadratic forms over complete discretely valued fields

David Leep
Department of Mathematics
University of Kentucky

The theory of quadratic forms over a complete discretely valued field K is well understood and rather easy when the residue field has characteristic different from 2. The standard examples are p-adic fields from number theory. This includes the fields Q_p and finite extensions of Q_p. When the residue field of K has characteristic 2, many results are also known but the proofs tend to be more difficult and sometimes a bit ad hoc. This is the case even for the field Q_2 and finite extensions of Q_2.

This talk presents a new method to develop the theory that greatly simplifies the presentation when the residue field of K has characteristic 2. For p-adic fields, we show how this leads to the standard results in quadratic form theory, including the computation of the u-invariant and the proof that the Hilbert symbol is nondegenerate.

Kadison's Pythagorean Theorem and Essential Codimension

Jireh Loreaux
Department of Mathematics and Statistics
Southern Illinois University Edwardsville
Victor Kaftal
Department of Mathematical Sciences
University of Cincinnati

In 2002, Richard Kadison published two papers in which he established an association between the diagonal of the matrix of an orthogonal projection and the Pythagorean Theorem. He pushed this abstraction of the Pythagorean Theorem quite far in that he completely characterized the diagonals of all orthogonal projections, even those acting on an infinite dimensional Hilbert space. In doing so, Kadison uncovered an unexpected integer that arises out of certain diagonals, and for quite some time the origin of this integer was somewhat mysterious. In this talk, we will explain the connection between diagonals of projections and the Pythagorean Theorem as demonstrated by Kadison, and then we present recent work with V. Kaftal which explains the origin of the integer in Kadison's theorem.

Some conjectural properties of coefficients of cyclotomic polynomials

Bogdan Petrenko
Eastern Illinois University

The goal of this talk is to interest the audience in some puzzling experimental observations about the asymptotic behavior of coefficients of cyclotomic polynomials. It is well known that any integer is a coefficient of some cyclotomic polynomial; we extend this result by showing that given n,all integers from -n to n appear as coefficients of some cyclotomic polynomial. Moreover, we observe an intriguing symmetry between the occurrence of positive and negative coefficients of cyclotomic polynomials. When various families of coefficients of cyclotomic polynomials are plotted on the computer screen, the resulting pictures appear "asymptotically almost symmetric". At present, we do not have any theoretical explanation of this perceived behavior of the coefficients. This talk is based on my joint work in progress with Brett Haines (Wolfram Research), Marcin Mazur (Binghamton University), and William Tyler Reynolds (University of Iowa).

The multiplicity one theorem for paramodular forms

Ralf Schmidt
Department of Mathematics
University of Oklahoma

Classical modular forms enjoy the *strong multiplicity one* property, which states that a newform is determined by almost all of its Hecke eigenvalues. This was proved by Atkin and Lehner in 1970. For higher rank modular forms, i.e., Siegel modular forms, the analogous theorem fails. In this talk we will explain why the strong multiplicity one property still holds for *paramodular forms*, an important class of Siegel modular forms of degree 2. Such forms have gained increasing importance in recent years due to their occurrence in the *paramodular conjecture*, a generalization of Shimura-Taniyama-Weil to rank 2.

Local Langlands Correspondence for Asai L and Epsilon Factors

Daniel Shankman
Department of Mathematics
Purdue University

Let E/F be a quadratic extension of p-adic fields. The local Langlands correspondence establishes a bijection between n-dimensional Frobenius semisimple representations of the Weil-Deligne group of E and smooth, irreducible representations of GL(n,E). We reinterpret this bijection in the setting of the Weil restriction of scalars Res(GL(n),E/F), and show that the Asai L-function and epsilon factor on the analytic side match up with the expected Artin L-function and epsilon factor on the Galois side.

On construction of automorphic representations via theta lifting.

Shuichiro Takeda
University of Missouri, Columbia

The theory of automorphic representations has been one of the central themes of contemporary mathematics. One of the major questions is a very simple question on how to construct automorphic representations in meaningful ways. One of the technologies for contracting automorphic representations is known as the method of theta lifting, which allows one to construct cuspidal automorphic representations of one reductive group out of those on another. In this talk, we will give an overview of this theory and the result obtained by Gan, Qiu and the speaker.