2024 Langenhop Lecture and SIU Mathematics Conference

School of Mathematical & Statistical Sciences
Southern Illinois University Carbondale Carbondale,
Illinois, USA

May 16–17, 2024

Welcome Address

After his retirement from SIU, Prof. Carl Langenhop gave financial support for a series of public lectures in the mathematical and statistical sciences. In more recent years, this lecture series has expanded into a conference, and it is always a great joy to have so many great friends and colleagues come to visit us for a few days.

Thank you for coming! We hope that you will enjoy your time here and that this meeting will be the site of many fruitful conversations.

“Diophantine equations” in partition functions

Ken Ono
University of Virginia

Abstract. The study of Diophantine equations has a long history, including the Pellian equations and the insidious Fermat equations. Here we take a particular look at a different kind of equation, “Diophantine equations” involving integer partition functions. Here we will present and announce results which will tie together many themes, from Hilbert’s Tenth Problem, to modular forms, and ultimately to the surprising realization that all functions, known as quasimodular forms, are partition theoretic in origin in the sense of Major MacMahon.

L - Functions and Langlands Program

Freydoon Shahidi
Department of Mathematics
Purdue University

Abstract: We introduce the notion of L-functions through infiniteness of primes and Euler products and discuss Eisenstein series which led Langlands to the definition of his L-functions. We then define Maass forms and what automorphic forms are. This allows us to introduce Langlands program and Functoriality Principle and success so far in proving them by means of L-functions and what the path forward may be. This talk is directed towards general mathematical audiences with little background on the subject.

Coloring Trivalent Graphs, Penrose State Sums and Virtual Knot Theory

Louis H Kauffman
Department of Mathematics
University of Illinois at Chicago 851 South Morgan Street
Chicago, Illinois 60607-7045

Abstract We begin by discussing the Penrose method for counting the num-ber of three colorings of a trivalent graph (with three distinct colors at a vertex). We show how to generalize the Penrose state summation to handle non-planar graphs and we generalize the notion of coloring to n-colorings of perfect matching graphs associated with a trivalent graph. We then show how these structures can be rewritten in terms of link diagrams and how there are significant generalizations of the Kauffman bracket polynomial associated with these Penrose generalizations. These ideas lead to the formulation of a multiple virtual knot theory and to other invariants in that theory (of quan-dle type) and to many questions and examples that we shall discuss. The talk will be self-contained.

Area of Specialization: Knot Theory, Combinatorics, Quantum Information

The Topological Symmetry Groups of the Petersen and Heawood Families

Robin Wilson
Department of Mathematics
Loyola Marymount University

The topological symmetry group of an embedding Γ of an abstract graph γ in S^3 is the group of automorphisms of γ which can be realized by homeomor-phisms of the pair (S^3, Γ). While motivated by questions about symmetries of molecules in space, the study of topological symmetry groups of graphs embedded in the 3-sphere can be thought of as a generalization of the study of symmetries of knots and links. In this talk we give a brief introduction to the study of spatial graphs and discuss recent results classifying topological symmetry groups for the Petersen and Heawood families of graphs.

Area of Specialization: Low-dimensional topology

Variants of Lehmer's Conjecture

Stephen DeBacker
Department of Mathematics
University of Michigan

Abstract Modular forms are generating functions of important quantities in arithmetic geometry, combinatorics, number theory, and physics. Despite many deep developments in the arit hmetic geometric and analytic aspects (e.g. Deligne's proof of the Weil Conjectures, the development of Galois represen- tations, Birch and Swinnerton-Dyer Conjecture, to name a few), some of the seminal questions about them remain open. Perhaps the most prominent of these is Lehmer's Conjecture on the nonvanishing of modular form coefficients such as Ramanujan's tau-function. In joint work with J. Balakrishnan, W. Craig, and W.-L. Tsai, the speaker has obtained the first results that establish that many integers are never modular form coefficients.

Preliminary results on the Steinberg module of the Braid Group

Nathan Broaddus
Department of Mathematics
Ohio State University

The Steinberg module of the braid group is the dualizing module for the braid group as well as the one non-trivial reduced homology group of the curve complex for the punctured disk. As such it is an important cohomo-logical object associated with the braid group. I will discuss recent joint work with L.-K. Lauderdale, E. Lawrence, A. Nu'Man and R. Wilson in which we investigate a presentation for the Steinberg modules of braid groups with low braid index. This work received generous support from the 2021 ADJOINT program at MSRI.

Area of Specialization: Steinberg Module of the Braid Group

Steinberg modules for low braid index

Anisah Nu'Man
Mathematics Department
Spelman College

I will introduce the braid group and discuss ongoing work on a presenta-tion of its Steinberg module. While the motivation for our work comes from group cohomology of the braid group, the computations that I will discuss will be very concrete involving some fun surface combinatorics. This work is joint with N. Broaddus, L.-K. Lauderdale, E. Lawrence and R. Wilson and was supported by the 2021 ADJOINT program at MSRI.

Area of Specialization: Steinberg Module of the Braid Group

Symmetric functions via a class of Markov chains

Phillip Feinsilver
Southern Illinois University Carbondale

Abstract. A class of Markov chains having a ladder structure is used to find relations among symmetric functions. This gives a probabilistic meaning to some familiar symmetric functions.

Symmetric Functions via Recurrences

Phillip Feinsilver
Southern Illinois University Carbondale

Abstract. We show the connections between linear recurrences and symmetric functions.
In particular, this approach leads to an interesting formula for Schur functions.

Mock Alexander Polynomials

Louis H Kauffman, UIC
Department of Mathematics
University of Illinois at Chicago 851 South Morgan Street
Chicago, Illinois 60607-7045

Abstract. This talk will discuss generalizations of the Alexander-Conway polynomial to starred knots, knotoids and knots in thickened surfaces. These generalizations use state summations that can be expressed in terms of permanents of matrices associated with the diagrams of the starreed entities. These state summations generalize the structures in the author's monograph Formal Knot Theory that originally apply to the Alexander-Conway polynomial. Thus we create analogs of the Alexander-Conway polynomial via state summation and study these Mock Alexander Polynomials in a number of contexts. In many contexts, Mock Alexander Polynomials can detect chirality. We will discuss our present state of knowledge and the conjectures that we are pursuing about them.
This talk is joint work with Neslihan Gugumcu.

Constructing new topological invariants of manifolds, differential geometry aspects

Robert Owczarek, University of New Mexico

Beginning from the work of Albert Schwarz and Edward Witten, one of the approaches to constructing topological invariants of manifolds emerged, based on quantum field theories that are independent of the metric. One such quantum field theory, based on Chern-Simons action for a gauge field based on the SU(2) gauge group was particularly successful in defining invariants related to Ray-Singer torsion for the 3D manifolds, and, when traces of holonomies along embedded circles (that might be knotted and linked) were added, invariants of knots and links of the kind of Jones polynomial could be defined as well, and for more general gauge groups (compact), some more general invariants known as Witten-Reshetikhin-Turaev could be also defined.

Does this exhaust the power of the metric-independent quantum field theory-based approach to invariants of manifolds?

Recently, we (Hanna Makaruk and I) started looking at quantum field theories based on classical field theories formulated by Kijowski and Sławianowski as potential sources of new invariants.

In this talk, I take a step back and consider the issue from the point of view of Cartan's philosophy of differential geometry, in which metric and affine connection are treated as independent fields. The Kijowski and Sławianowski theories will be looked at from this point of view and some remarks on the developments of the topological quantum field theories for the manifolds of general dimension, and in dimension three will be made.

Abstractions on Koopman theory of dynamical systems

Robert Owczarek
University of New Mexico

Koopman operator theory, invented in the 30s of the 20th recently became a very valuable tool in studying dynamical systems, both with continuous and discrete time. The reason for this is the recent explosion of data-driven studies of the evolution of structures described in terms of dynamical systems.

The main idea is to consider spaces of functions of the variables (in general, elements of a smooth manifold) described in terms of dynamical systems, which are then “observables” associated with it, somewhat in the spirit of quantum mechanics. Under mild assumptions on the right-hand side of the dynamical system, one can prove an existence and uniqueness theorem, which allows the introduction of a semi-group of propagators acting on an initial condition and giving the solution at a given later time. The Koopman operator is an intertwiner for the action of the semi-group on the functions, and as such is a linear operator. Linearity is a very exciting feature and thus the spectral analysis of

Koopman operators became an industry of a kind leading to important applications. However, there are more exciting features of the Koopman operators, which I would like to talk about, as well as some generalizations of the Koopman operator naturally appearing when discussing the operator at an abstract approach I am going to introduce.

Generalizations of Arnold's Version of Euler's Theorem for Matrices

Bogdan Petrenko
Department of Mathematics & Computer Science
Eastern Illinois University

I plan discuss my two joint work with Marcin Mazur (SUNY Binghamton). We have proved the following results:

- For a square matrix A with integer entries, a prime number p, and a positive integer k, the characteristic polynomials of the matrices A^{p^k} and $A^{p^{k-1}}$ are congruent modulo p^k.

- Let $\Phi(n) = \dfrac{1}{2}\left(\varphi(n) + [\mathrm{SL}_2(\mathbb{Z}) : \Gamma_0(n)]\right)$ and A a 2-by-2 integer matrix, then for any $n \geq 2$, the characteristic polynomials of $A^{\Phi(n)}$ and $A^{\Phi(n)-\varphi(n)}$ are congruent modulo n.

- For infinitely many n that are products of distinct prime numbers, and for any $k \geq 3$, there exists a k-by-k integer matrix A such that the characteristic polynomials of $A^{m+\varphi(n)}$ and A^m are not congruent modulo n for any $m \geq 1$.

Complex Zeon Polynomials

G. Stacey Staples
Department of Mathematics & Statistics
Southern Illinois University Edwardsville

Letting the finite subsets of positive integers be denoted by $[\mathbb{N}]^{<\omega}$, the *complex zeon algebra*, denoted by $\mathbb{C}\mathfrak{Z}$, has a canonical basis of the form $\{\zeta_I : I \in [\mathbb{N}]^{<\omega}\}$, where $\zeta_\varnothing = 1$ and the product of two basis elements satisfies the following:

$$\zeta_I\zeta_J = \begin{cases} \zeta_{I\cup J} & I \cap J = \varnothing, \\ 0 & \text{otherwise.} \end{cases}$$

An element $u \in \mathbb{C}\mathfrak{Z}$ has canonical expansion $u = \sum_I u_I\zeta_I$, where each I is a finite subset of $\mathbb{N}$, $u_I \in \mathbb{C}$, and only finitely many of the coefficients u_I are nonzero.

When considering polynomials with complex zeon coefficients, null-square properties of the zeon generators lead to interesting properties and combinatorial applications.

For example, a cubic polynomial with zeon coefficients may have three "spectrally simple" zeros, infinitely many zeros, or no zeros at all. A classification of zeros is possible via an extension of the cubic discriminant to zeon polynomials.

Further, where every nonzero complex number has exactly k distinct kth roots, a nonzero complex zeon monomial $\alpha\,\zeta_I$ has infinitely many kth roots, provided $|I| \geq k$. These roots can be grouped into some easily characterized "fundamental" equivalence classes whose cardinalities are multiples of Stirling numbers of the second kind.

Turning to characteristic polynomials of zeon matrices, eigenvalues are natural objects of interest. Square zeon matrices of order m naturally represent $\mathbb{C}\mathfrak{Z}$-linear operators on the module $\mathbb{C}\mathfrak{Z}^m$. In particular, eigenvalues of the zeon combinatorial Laplacian of a finite graph provide information about the graph's cycles.

Area of Specialization: Algebra; Combinatorics

Enhanced power graphs of finite Moufang loops

Ronald White
Southern Illinois University Carbondale

Abstract: Extensive research has been conducted on enhanced power graphs within the realm of finite groups. In this study, we extend our focus beyond finite groups to the domain of finite Moufang loops. This presentation will provide an introduction to the fundamental concepts of loops and enhanced power graphs. The exploration will culminate in the presentation of a result, accompanied by a minimal example.

Hilbert modular forms from orthogonal modular forms on quaternary lattices

Eran Assaf & John Voight
Department of Mathematics
Dartmouth College
Dan Fretwell
Department of Mathematics and Statistics
University of Lancaster
Colin Ingalls
School of Mathematics and Statistics
Carleton University
Adam Logan
Institute for Computational and Experimental Research in Mathematics Brown University
Spencer Secord
Department of Pure Mathematics
University of Waterloo

Let F be a totally real field with ring of integers $\mathbb{Z}_F$, let (V, Q) be a to-tally positive definite quadratic space over F with $\dim V = 4$, and let $\Lambda \subseteq V$ be a $\mathbb{Z}_F$-lattice. The set of functions on $\mathrm{cls}(\Lambda)$, can be viewed as a space of modular forms for the group $O(V)$.

Initially, this space was studied using theta series attached to Λ. When $F = \mathbb{Q}$, this approach, taken by Brandt and Eichler, focused on the case where $(V, Q) = (B, \mathrm{nrd})$ is the reduced norm form on a quaternion alge-bra B, and Λ is a maximal order. In particular, such forms always have $\mathrm{disc}\,\Lambda = N^2$, and relate to pairs of modular forms $(f, g) \in S_2(N)$, as ob-served by Böcherer and Schulze-Pillot.

In this talk, we explicitly determine the relationship between Hilbert modular forms $f \in S_{k_1,k_2}(\mathfrak{N}\mathbb{Z}_K)$ where K/F is a quadratic étale algebra, and modular forms for Λ with values in a weight representation $\rho : O(V) \to W$, with precise level structure and weight. We present algorithmic applications, and by connecting our results to the theory of theta correspondence, present an application to the non-vanishing of theta maps.

Area of Specialization: Number Theory

Universal Binary Quadratic Forms

Archisman Bhattacharjee
Department of Mathematics
Louisiana State University

Abstract: In this talk, we classify binary quadratic forms that represent all the integers. Our development relies mainly on the results by Duncan. A Buell on the representation of specific integers by specific binary quadratic forms. We also use the results by Alaca, Alaca, and Williams on binary quadratic forms representing an arithmetic progression. We show that the forms we are interested in must have discriminant 1, which helps further in the classification.

Can we recover an integral quadratic forms by representing all its proper subforms?

Wai Kiu Chan
Department of Mathematics and Computer Science
Wesleyan University

An integral quadratic form is said to be represented by an integral quadratic form f if it is obtained from f by a linear change of variables. Such a quadratic form is called a proper subform of f if the change of variables is not invertible. If g is another integral quadratic form which represents all proper subforms of f, does g represent f? In this talk, I will discuss an answer to this question, highlighting the contrast between the indefinite and the definite cases.

This is a joint work with Byeong-Kweon Oh at Seoul

Can we recover an integral quadratic forms by representing all its proper subforms?

Wai Kiu Chan
Department of Mathematics and Computer Science
Wesleyan University

An integral quadratic form is said to be represented by an integral quadratic form f if it is obtained from f by a linear change of variables. Such a quadratic form is called a proper subform of f if the change of variables is not invertible. If g is another integral quadratic form which represents all proper subforms of f, does g represent f? In this talk, I will discuss an answer to this question, highlighting the contrast between the indefinite and the definite cases.

This is a joint work with Byeong-Kweon Oh at Seoul National University. *Area of Specialization: Integral quadratic forms*

On Lattice Extensions

Maxwell Forst
Mathematics & Statistics Department University of Minnesota Duluth

A lattice in a Euclidean space is a free Z-module. A lattice M is said to be an extension of a sublattice L of smaller rank if the intersection of M and the subspace spanned by L is equal to L. We initiate a systematic study of the geometry of lattice extensions, proving the existence of a small-determinant integral extension of a given integer lattice, as well as extensions with con-trolled successive minima and covering radius. Time permitting, we will also discuss some interesting arithmetic properties of deep holes of planar lattices, which are points in the space furthest removed from the lattice. Joint work with Lenny Fukshansky.

Area of Specialization: Number Theory

Deep hole lattices and isogenies of elliptic curves

Lenny Fukshansky
Department of Mathematical Sciences
Claremont McKenna College

For a lattice L in the plane, we define the affiliated deep hole lattice $H(L)$ to be spanned by a shortest vector of L and the furthest removed vector from the lattice contained in the triangle with sides corresponding to the shortest basis vectors. We study the geometric and arithmetic properties of deep hole lattices, which turn out to be quite interesting. In particular, we construct sequences of deep hole lattices corresponding to elliptic curves over a fixed number field. In the case of CM elliptic curves, we prove that all elliptic curves generated by this sequence are isogenous to each other and produce bounds on the degree of isogeny. Finally, we produce a counting estimate for the planar lattices with a prescribed deep hole lattice. Joint work with Pavel Guerzhoy and Tanis Nielsen.

Area of Specialization: Number Theory

Primitive Elements in Number Fields and Diophantine Avoidance

Sehun Jeong
Institute of Mathematical Sciences
Claremont Graduate University

The famous primitive element theorem states that every number field K is of the form $Q(a)$ for some element a in K, called a primitive element. In fact, it is clear from the proof of this theorem that not only there are infinitely many such primitive elements in K, but in fact most elements in K are prim-itive. This observation raises the question about finding a primitive element of small "size", where the standard way of measuring size is with the use of a height function. We discuss some conjectures and known results in this direction, as well as some of our recent work on a variation of this problem which includes some additional avoidance conditions. Joint work with Lenny Fukshansky.

Area of Specialization: Number Theory

Universality of congruent quadratic forms

Koustav Mondal
Department of Mathematics
Louisiana State University

In this talk, we consider representations of positive integers as sums of four distinct generalized triangular numbers, which is an extended version of Sun's conjectures of universal generalized triangular numbers. A known method of dealing with this is to convert the quadratic polynomial into a quadratic form with congruence conditions and study the associated theta series. We will use some examples to demonstrate this method.

Area of Specialization: theta series arising from quadratic forms

Pythagoras numbers in biquadratic fields

Ester Sgallová
Department of Mathematics
Michigan State University

In this talk, we will focus on quadratic forms over quadratic and bi-quadratic fields and their connection to the Pythagoras number, i.e. the smallest number n such that any sum of squares could be written as a sum of n squares. We will examine the Pythagoras number of the ring of integers in a totally real biquadratic number field. We will see that the known upper bound 7 is attained in a large and natural infinite family of such fields. Still, there exist infinitely many biquadratic fields which have smaller Pythagoras number.

Area of Specialization: Integral Quadratic Forms, Number Theory

Recoverability of the lattice $\langle 1 \rangle \perp A_n$ for any $n \geq 2$

Felipe Valdes Gonzalez
Department of Mathematics
Wesleyan University

Let f be a positive definite quadratic form over the ring of integers. We say f is recoverable if any form g that represents all proper sub-forms of f also represents f. It was proved by *Chan-Oh* that any indefinite quadratic form is recoverable, but any positive definite indecomposable quadratic forms is not recoverable. Little is known about positive definite decomposable forms. We define the root

lattice A_n as the set of elements in the lattice Z^{n+1} whose coordinates add up to 0. In this talk, I will present a proof that the decomposable positive definite lattice $\langle 1 \rangle \perp A_n$ is not recoverable for any $n \geq 2$.

Area of Specialization: Quadratic Forms, Integral Quadratic forms theory

A Flow approach to Prescribing the Q'-curvature on Pseudo-Einstein 3-Manifolds

Ali Maalaoui –
Department of Mathematics
Clark University

In this talk I will present the problem of prescribing the Q'-curvature on compact CR 3-manifolds. To address the question, we use a flow approach that is adapted to the non-local nature of the problem. We show that under natural assumptions on the prescribed quantity, the flow converges polynomially. For the critical case corresponding to $\int_M Q' \, dv_\theta = 16\pi^2$ one needs to use an improved Moser-Trudinger inequality to insure the convergence of the flow.

This work is in collaboration with Vittorio Martino.

Area of Specialization: Partial Differential Equations/ Geometric Analysis

Adams inequalities with exact growth conditions on metric measure spaces

Carlo Morpurgo
Department of Mathematics
University of Missouri, Columbia
USA

Liuyu Qin
Department of Mathematics, Statistics
Hunan University of Finance and Economics
Changsha, Hunan
China

In this talk I will give a brief survey of recent results on Moser-Trudinger inequalities with exact growth conditions on Euclidean spaces. I will then present the latest new developments (in joint work with Liuyu Qin) on met-ric measure spaces, including in particular $\mathbb{R}^n$, the Heisenberg group, and Hadamard manifolds. I will also discuss a PDE application on $\mathbb{R}^n$.

Area of Specialization: Analysis

Morse Theory and the resonant T-curvature equation

Cheikh Birahim Ndiaye
Department of Mathematics
Howard University

In this talk, we present new existence results for the resonant T -curvature equation on compact 4-manifolds with boundary. We first present a complete and explicit classification of the solutions of the problem in the case of the unit ball. Next, using our classification results combined with Bubbling Anal-ysis and Morse Theory, we discuss the case of arbitrary compact 4-manifold presenting existence, compactness, and Leray-Schauder degree counting for-mula.

Area of Specialization: Partial Differential Equations

4th Order Stochastic PDEs with Q-Regular Space-Time Noise

Henri Schurz
Department of Mathematics
Southern Illinois University, Carbondale, IL

In this talk, we present major tools of analysis of nonlinear and linear ran-dom beam equations. This relates to nonrotating and rotating beams with displacement $U = U(x, t, \omega)$ satisfying a **(nonlinear) stochastic PDEs of 4th order** with homogeneous boundary conditions (for simplicity of pre-sentation). Fourier series analysis can be successfully exploited to conduct a **fairly thorough analysis of random beam problems** with adap-tive or nonrandom boundary conditions of Dirchlet, Neumann or similar linear boundary conditions. The techniques of Lyapunov functionals and energy estimates are used. Qualitative assertions on existence and unique-ness, measurability, Hölder-continuity (regularlity), uniform moment bound-edness, asymptotic stability under martingale-type space-time noise can be made for

approximated L^2-solutions in appropriate Hilbert spaces in a straightforward manner (without discretizing the underlying state-space). A new **trace formula for the total energy functional** with additive noise has been found by us, published in JCAM (2011) and the studies are extended to rotating beam problem in IJACM (2024). Finally, we make remarks how to implement numerical approximations by nonstandard numerical methods in order to carry out simulations for "interested practicioner".

The talk is based on **joint work with Prof. Dr. Boris Belinskiy** (UT at Chattanooga, Tennessee) - an esteemed specialist in control theory and PDEs.

Area of Specialization: Stochastic PDEs, Random Vibrations, Engineering

Existence and non-existence results for elliptic systems on bounded and unbounded domains

John Villavert
School of Mathematical and Statistical Sciences
University of Texas, RGV

In this talk, we shall focus on the non-negative solutions to a broad family of elliptic problems in the setting of the whole space or in bounded star-shaped domains. The type of equations and systems within this fam-ily include classical ones arising in finding the best constant in functional inequalities and curvature problems from conformal geometry. The model example is perhaps the scalar equation

$$div(a(x)\nabla u) + f(x, u) = 0, u \geq 0, \text{in } \Omega,$$

where we place suitable conditions on the weight $a(x)$, the nonlinearity f and the domain $\Omega \subset R^n$. In particular, under certain growth conditions on the source nonlinearities and geometric assumptions on the domain, we can establish various existence and non-existence results, including Liouville-type theorems. We will describe these main results and outline the techniques for generating the results, which center around moving plane methods and degree theoretic shooting methods.

Area of Specialization: Partial Differential Equations

Asymptotic Analysis and Uniqueness of blowup solutions of non-quantized singular mean field equations

Lei Zhang
Department of Mathematics
University of Florida

For singular mean field equations defined on a compact Riemann sur-face, we prove the uniqueness of bubbling solutions as far as blowup points are either regular points or non-quantized singular sources. In particular the uniqueness result covers the most general case extending or improving all pre-vious works. For example, unlike previous results, we drop the assumption of singular sources being critical points of a suitably defined Kirchoff-Routh type functional. Our argument is based on refined estimates, robust and flexible enough to be applied to a wide range of problems requiring a delicate blowup analysis. In particular we come up with a major simplification of previous uniqueness proofs. This is a joint work with Daniele Bartolucci and Wen Yang.

Area of Specialization: Nonlinear Partial Differential Equations

p-Modular Representations of $SL3$ over a Finite Field

Devjani Basu
School of Mathematical and Statistical Sciences
Southern Illinois University Carbondale

The modular representations of a group G are the representations over a vector space over a field of nonzero characteristic ℓ, when ℓ divides the (pro-) order of the group G. As a consequence, the semisimplicity of the group algebra is lost. For the case, $\ell = p$, to study the p-modular representations

of $SL3(\mathbb{F}_q)$ in particular, and $G = SL_n(\mathbb{F}_q)$ in general, we first concentrate on those modules (or *vector spaces*) which are realized within the group algebra

$\mathbb{F}G$, where $\mathbb{F}$ is the algebraic closure of $\mathbb{F}_q$. A natural approach to study the simple $\mathbb{F}G$-modules is to describe them as a restriction of simple modules for the algebraic group $SL_n(\mathbb{F})$ to that of $SL3(\mathbb{F}_q)$.

As a finite group of Lie type, $SL3(\mathbb{F}_q)$, is a close relative of the groups of rational points of algebraic groups defined over $\mathbb{F}_q$. Therefore, we have adapted the methodology developed by Cédric Bonnafé for $SL2(\mathbb{F}_q)$ to de-scribe the p-modular representations of $SL3(\mathbb{F}_q)$ *explicitly.*

Area of Specialization: Representation Theory

The Adams conjecture and intersections of local Arthur packets Alexander Hazeltine
Department of Mathematics
University of Michigan

The Adams conjecture states that the local theta correspondence sends a local Arthur packet to another local Arthur packet. Mœglin confirmed the conjecture when lifting to groups of sufficiently high rank and also showed that it fails in low rank. Recently, Baki ́c and Hanzer described when the Adams conjecture holds in low rank for a representation in a fixed local Arthur packet. However, a representation may lie in many local Arthur pack-ets and each gives a minimal rank for which the Adams conjecture holds. In this talk, we discuss the relation between the Adams conjecture and intersec-tions of local Arthur packets. This approach leads to a solution for a failure of the Adams conjecture.

Area of Specialization: Number Theory

Title: Functorial Descent in the Exceptional Groups

Joseph Hundley
Department of Mathematics
University at Buffalo

Abstract: In this talk, I will discuss some recent attempts (one joint with Liu and another with Ginzburg) to extend the method of functorial descent, developed by Ginzburg, Rallis and Soudry in the classical groups, to the exceptional groups, and challenges and new phenomena which emerge in these attempts.

Dagger Groups and p-adic distribution algebras

Aranya Lahiri
Department of Mathematics
University of California San Diego

Abstract: p-valued groups have played an important role in the study of locally analytic representations since their introduction by Schneider-Teitelbaum. In this talk I will outline how to attach a dagger group (a group object in the category of dagger spaces as introduced by Grosse-Klonne) to a p-saturated group. I will further introduce the concept of overconvergent functions and overconvergent distribution algebras on such a group. And finally continuing the lineage of a result by Schneider-Teitelbaum in the locally analytic setup I will show that the overconvergent distribution algebra of a uniform pro-p group is Frechet-Stein.

Recent progress on certain problems related to local Arthur packets of classical groups

Baiying Liu
Department of Mathematics
Purdue University

Abstract: In this talk, I will introduce recent progress on certain problems related to local Arthur packets of classical groups. First, I will introduce a joint work with Freydoon Shahidi towards Jiang's conjecture on the wave front sets of representations in local Arthur packets of classical groups, which is a natural generalization of Shahidi's conjecture, confirming the relation between the structure of wave front sets and the local Arthur parameters. Then, I will introduce a joint work with Alexander Hazeltine and Chi-Heng Lo on the intersection problem of local Arthur packets for symplectic and split odd special orthogonal groups, with applications to the Enhanced Shahidi's conjecture, the closure relation conjecture, and the conjectures of Clozel on unramified representations and on unramified components of automorphic representations. This intersection problem also has been worked out independently at the same time by Hiraku Atobe. In a recent joint work with Alexander Hazeltine, Chi-Heng Lo, and Freydoon Shahidi, we made an upper bound conjecture on wavefront sets of admissible representations of connected reductive groups. If time permits, I will also briefly introduce this conjecture and our recent progress towards it.

An upper bound for wavefront sets of admissible representations of p-adic groups

Chi-Heng Lo
Department of Mathematics
Purdue University

Wavefront set is a fundamental invariant for representations of p-adic groups. An interesting and widely open question is how to describe it in terms of the classification of irreducible representations. In a joint work with Alexander Hazeltine, Baiying Liu and Freydoon Shahidi, we propose a new conjecture on an upper bound for the wavefront sets of admissible represen-tations of p-adic groups. In this talk, I will introduce this conjecture and our work on it. More precisely, I will discuss the equivalence between our new conjecture and Jiangs Conjecture for pure inner forms of classical groups, which predicts an upper bound for the wavefront set of each representation of Arthur type in terms of the local Arthur packets it lies in. I will also discuss the reduction of these conjectures to the case for Aubert-Zelevinsky involution of discrete series representations.

Area of Specialization: Representation Theory and Automorphic Forms

Weyl's Law for Arbitrary Archimedean Type

Ayan Maiti
Department of Mathematics
Purdue University

In this paper we generalize the work of Lindenstrauss and Venkatesh es-tablishing Weyl's Law for cusp forms from the spherical spectrum to arbitrary archimedean type. Weyl's law for the spherical spectrum gives an asymp-totic formula for the number of cusp forms that are bi-K_∞ invariant in terms of eigenvalue T of the Laplacian. We prove an analogous asymptotic holds for cusp forms with archimedean type τ, where the main term is multiplied by dim τ. While in the spherical case, the surjectivity of the Satake Map was used, in the more general case that is not available and we use Arthur's Paley-Wiener theorem and multipliers. In this talk I will provide some back-grounds, previous results and methods to prove the estimate.

Area of Specialization: Automorphic forms and Representations

Projective smooth representations modulo p

Claus Sorensen
Department of Mathematics
University of California San Diego

Abstract: This talk will be borderline colloquial and geared towards people from other areas. I will talk about smooth mod p representations of p-adic Lie groups.
Such categories of modular representations play a central role in the p-adic Langlands program. In sharp contrast to the complex case, as well as the case of finite groups, these categories typically do not have any projective objects (other than zero). For reductive groups this is a byproduct of a stronger result on the vanishing range of the derived functors of full smooth induction from a congruence subgroup. The talk is based on joint work with Peter Schneider.

Resolutions for locally analytic representations

Shishir Agrawal
Department of Mathematics
University of California at San Diego
Matthias Strauch
Department of Mathematics
Indiana University

We explain joint work with Shishir Agrawal on resolutions for locally analytic representations. These are representations of p-adic groups on topological vector spaces over the p-adic numbers. Smooth representations are examples of locally analytic representations, but the class of locally analytic representations is much larger. These representations have applications in the area of p-adic automorphic forms.

Let G be a p-adic reductive group. Following the groundbreaking work of Schneider and Stuhler we construct complexes by means of analytic vectors for a family of compact-open subgroups indexed by the simplices of the Bruhat-Tits building of G. The exactness of these complexes depends on an analogue of the Bernstein-Borel-Matsumoto theorem for locally analytic representations which is currently still conjectural.

In order to have available a good homological framework, we work in the category of solid locally analytic representations as developed by Rodrigues Jacinto and Rodríguez Camargo. Our work has also been inspired by J. Kohlhaase's paper on the cohomology of locally analytic representations.

Area of Specialization: Representation theory

Asymptotic Analysis and Uniqueness of blowup solutions of non-quantized singular mean field equations

Lei Zhang
Department of Mathematics
University of Florida

For singular mean field equations defined on a compact Riemann sur-face, we prove the uniqueness of bubbling solutions as far as blowup points are either regular points or non-quantized singular sources. In particular the uniqueness result covers the most general case extending or improving all pre-vious works. For example, unlike previous results, we drop the assumption of singular sources being critical points of a suitably defined Kirchoff-Routh type functional. Our argument is based on refined estimates, robust and flexible enough to be applied to a wide range of problems requiring a delicate blowup analysis. In particular we come up with a major simplification of previous uniqueness proofs. This is a joint work with Daniele Bartolucci and Wen Yang.

Area of Specialization: Nonlinear Partial Differential Equations