
# 2024 Langenhop Lecture and SIU Mathematics Conference

# Steinberg Module of the Braid Group

School of Mathematical & Statistical Sciences
Southern Illinois University Carbondale
Carbondale, Illinois, USA
May 16–17, 2024

# Welcome Address

After his retirement from SIU, Prof. Carl Langenhop gave financial support for a series of public lectures in the mathematical and statistical sciences. In more recent years, this lecture series has expanded into a conference, and it is always a great joy to have so many great friends and colleagues come to visit us for a few days.

Thank you for coming! We hope that you will enjoy your time here and that this meeting will be the site of many fruitful conversations.

# “Diophantine equations” in partition functions

Ken Ono
University of Virginia

Abstract. The study of Diophantine equations has a long history, including the Pellian equations and the insidious Fermat equations. Here we take a particular look at a different kind of equation, “Diophantine equations” involving integer partition functions. Here we will present and announce results which will tie together many themes, from Hilbert’s Tenth Problem, to modular forms, and ultimately to the surprising realization that all functions, known as quasimodular forms, are partition theoretic in origin in the sense of Major MacMahon.

# L - Functions and Langlands Program

Freydoon Shahidi
Department of Mathematics
Purdue University

Abstract: We introduce the notion of L-functions through infiniteness of primes and Euler products and discuss Eisenstein series which led Langlands to the definition of his L-functions. We then define Maass forms and what automorphic forms are. This allows us to introduce Langlands program and Functoriality Principle and success so far in proving them by means of L-functions and what the path forward may be. This talk is directed towards general mathematical audiences with little background on the subject.

# Coloring Trivalent Graphs, Penrose State Sums and Virtual Knot Theory

Louis H Kauffman
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**Abstract** We begin by discussing the Penrose method for counting the num-ber of three colorings of a trivalent graph (with three distinct colors at a vertex). We show how to generalize the Penrose state summation to handle non-planar graphs and we generalize the notion of coloring to n-colorings of perfect matching graphs associated with a trivalent graph. We then show how these structures can be rewritten in terms of link diagrams and how there are significant generalizations of the Kauffman bracket polynomial associated with these Penrose generalizations. These ideas lead to the formulation of a multiple virtual knot theory and to other invariants in that theory (of quan-dle type) and to many questions and examples that we shall discuss. The talk will be self-contained.

*Area of Specialization: Knot Theory, Combinatorics, Quantum Information*

# The Topological Symmetry Groups of the Petersen and Heawood Families

Robin Wilson
Department of Mathematics
Loyola Marymount University

The topological symmetry group of an embedding $\Gamma$ of an abstract graph $\gamma$ in $S^3$ is the group of automorphisms of $\gamma$ which can be realized by homeomor-phisms of the pair $(S^3, \Gamma)$. While motivated by questions about symmetries of molecules in space, the study of topological symmetry groups of graphs embedded in the 3-sphere can be thought of as a generalization of the study of symmetries of knots and links. In this talk we give a brief introduction to the study of spatial graphs and discuss recent results classifying topological symmetry groups for the Petersen and Heawood families of graphs.

*Area of Specialization: Low-dimensional topology*

# Variants of Lehmer's Conjecture

Stephen DeBacker
Department of Mathematics
University of Michigan

**Abstract** Modular forms are generating functions of important quantities in arithmetic geometry, combinatorics, number theory, and physics. Despite many deep developments in the arit hmetic geometric and analytic aspects (e.g. Deligne's proof of the Weil Conjectures, the development of Galois represen- tations, Birch and Swinnerton-Dyer Conjecture, to name a few), some of the seminal questions about them remain open. Perhaps the most prominent of these is Lehmer's Conjecture on the nonvanishing of modular form coefficients such as Ramanujan's tau-function. In joint work with J. Balakrishnan, W. Craig, and W.-L. Tsai, the speaker has obtained the first results that establish that many integers are never modular form coefficients.

# Preliminary results on the Steinberg module of the Braid Group

Nathan Broaddus
Department of Mathematics
Ohio State University

The Steinberg module of the braid group is the dualizing module for the braid group as well as the one non-trivial reduced homology group of the curve complex for the punctured disk. As such it is an important cohomo-logical object associated with the braid group. I will discuss recent joint work with L.-K. Lauderdale, E. Lawrence, A. Nu'Man and R. Wilson in which we investigate a presentation for the Steinberg modules of braid groups with low braid index. This work received generous support from the 2021 ADJOINT program at MSRI.

*Area of Specialization: Steinberg Module of the Braid Group*

**Steinberg modules for low braid index**

Anisah Nu'Man
Mathematics Department
Spelman College

I will introduce the braid group and discuss ongoing work on a presenta-tion of its Steinberg module. While the motivation for our work comes from group cohomology of the braid group, the computations that I will discuss will be very concrete involving some fun surface combinatorics. This work is joint with N. Broaddus, L.-K. Lauderdale, E. Lawrence and R. Wilson and was supported by the 2021 ADJOINT program at MSRI.

*Area of Specialization: Steinberg Module of the Braid Group*